Minimal supersymmetric SU(5) model with nonrenormalizable operators: Seesaw mechanism and violation of flavour and CP

Francesca Borzumati[1,2] and Toshifumi Yamashita[3]

[1] *Department of Physics, National Taiwan University, Taipei 10617, Taiwan*
[2] *KEK Theory Division, Tsukuba, 305-0801 Japan*
[3] *Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

Flavour and CP violations that the neutrino-seesaw couplings of type I, II, III induce radiatively in the soft massive parameters of the minimal supersymmetric SU(5) model, made realistic by nonrenormalizable operators, are analyzed. Effective couplings are used to parametrize the couplings of renormalizable operators and of the corrections that nonrenormalizable ones provide at the tree level. It is found that for a limited, but sufficient accuracy in the calculations of such violations, it is possible to extend the picture of effective couplings to the quantum level, all the way to the cutoff scale. The arbitrariness introduced by nonrenormalizable operators is analyzed in detail. It is shown that it can be drastically reduced in the Yukawa sector if the effective Yukawa couplings involving colored triplet Higgs bosons are tuned to suppress the decay rate of the proton. In the supersymmetry-breaking sector the usual requirement of independence from flavour and field type for the mechanism of mediation of supersymmetry breaking is not sufficient to forbid arbitrary flavour and CP violations at the tree level. Special conditions to be added to this requirement, under which such violations can be avoided, are identified. Depending on how and whether these conditions are implemented, different phenomenological scenarios emerge. Flavour and CP violations of soft massive parameters induced by neutrino-seesaw couplings are discussed explicitly for the simplest scenario, in which no such violations are present at the tree level. Guidelines for studying them in other, less simple scenarios are given. Lists of all renormalization group equations needed for their calculations are provided for each of the three types of the seesaw mechanism, at all energies between the TeV scale and the Planck scale.

§1. Introduction and motivation

Among the existing proposals to solve the hierarchy problem of the Standard Model (SM), supersymmetry (SUSY) is still one of the most compelling. A solution to this problem without excessive tuning requires that the massive parameters that break SUSY softly are around the TeV scale, hereafter identified with the electroweak scale, $M_{\rm weak}$ or $\widetilde{m}$.

As is well known, if no restrictions are invoked for these parameters, flavour violations in the sfermion mass matrices, or sfermion flavour violations (sFVs), in general exist. In particular, off-diagonal elements in the chirality-conserving sectors of these matrices, and/or misalignments of the chirality-mixing sectors with the corresponding Yukawa matrices, may be nonnegligible. Then, loop diagrams with exchange of superpartners can give large contributions to flavour-changing-neutral-current (FCNC) processes. Since these are experimentally known to be rare, and not in disagreement with the SM predictions,[1)] it follows that sFVs must be rather small,[2)] or altogether absent, at least at one scale.

Even if vanishing at the cutoff scale, $M_{\rm cut}$, however, sFVs are in general nonvanishing at $M_{\rm weak}$, as they are induced at the quantum level by the SM Yukawa

couplings.[3)–7)] They are suppressed by a loop factor, but enhanced by the large logarithms of the ratios of $M_{\rm cut}$ and $M_{\rm weak}$, usually resummed through renormalization group (RG) techniques. Nevertheless, the squark flavour violations (sQFVs) obtained in this way in the minimal supersymmetric standard model (MSSM) are not particularly large[4),6)] (unless $\tan\beta$ is large[8)]), mainly because of the smallness of the off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $K_{\rm CKM}$, and the pattern of fermion masses.

Irrespective of the extensions made to solve the hierarchy problem, the leptonic sector requires also some enlargement of the originally proposed SM structure in order to accommodate neutrino masses. One way to proceed is to introduce the well-known seesaw mechanism, which is classified into three different types, I,[9)] II,[10)] and III,[11)] depending on which heavy fields are advocated for its mediation.[12)] Singlets of the SM, or right-handed neutrinos (RHNs) are used for the type I. All three types of seesaw mediators have couplings of $\mathcal{O}(1)$ with the lepton doublets if their scale, $M_{\rm ssw}$, is large, considerably above the range of energies at which direct detection is possible.

It is therefore very important to search for signals that can give information on these heavy fields. An obvious magnifying glass for them may be precisely their large Yukawa couplings to the left-handed leptons and the large mixing angles of the Maki-Nakagawa-Sakata (MNS) matrix, $K_{\rm MNS}$. These affect the RG flow of the soft SUSY-breaking parameters for sleptons,[13)] inducing slepton flavour violations (sLFVs), and consequently flavour-violating effects in the charged-lepton fermion sector. Thus, information on the seesaw fields can be hopefully gleaned through the study of flavour-violating processes in models in which sFVs at $M_{\rm cut}$ are vanishing (or under control, as in models with a Yukawa mediation of SUSY breaking[14)]). The existence of arbitrary tree-level flavour violations in the slepton mass parameters at this scale, even if relatively small, for example of $\mathcal{O}(10^{-2})$, can completely obscure the loop effects induced by the seesaw mechanism.

In this type of studies, a minimal number of parameters is commonly used to specify the soft SUSY-breaking terms at $M_{\rm cut}$: a common gaugino mass, $M_{1/2}$, a mass common to all scalars, with vanishing intergenerational mixings, $\widetilde{m}_0$, and two, in general, uncorrelated parameters, the bilinear and trilinear couplings B_0 and A_0, which guarantee the alignment of the bilinear and trilinear soft terms to the corresponding mass and Yukawa terms in the superpotential. This short list of parameters encapsulates the concept of universal boundary conditions for soft masses, which may be obtained in the context of minimal supergravity,[15),16)] or more generally, from a field- and flavour-blind mediation of SUSY breaking, such as gravity mediation, which is assumed throughout this paper.

If the MSSM with the seesaw mechanism is embedded in a SUSY SU(5) grand unified theory (GUT), the seesaw mediators interact with large Yukawa couplings not only with the SU(2)-doublet leptons, but with all components of the multiplets to which these leptons belong, that is, also with SU(2)-singlet down quarks. Thus, as pointed out by Baek *et al*[17)] and Moroi,[18)] in the minimal SUSY SU(5) (MSSU(5)) model with the seesaw of type I, these interactions can induce sQFVs at $M_{\rm weak}$ in the right-right down-squark sector, whereas those generated by the top Yukawa

coupling are in the left-left sector. Such sQFVs are related to sLFVs in a simple way. In addition, various independent phases are present in this model, only one of which corresponds to the CKM phase of the SM, when evolved at low scale. These GUT phases can also leave visible imprints in the superpartner mass matrices, during the RG flow from $M_{\rm cut}$ to the GUT scale, $M_{\rm GUT}$, if the soft SUSY-breaking parameters are real at $M_{\rm cut}$.[18)]

Both these facts have raised hopes that combined studies of flavour- and CP-violating processes in the quark and lepton sector may provide interesting avenues for detecting the presence of RHNs in the MSSU(5) model with universal soft terms at $M_{\rm cut}$. Indeed, considerable attention has been paid to the correlations of sQFVs and sLFVs in this model,[17)–23)] much less in the MSSU(5) model with the other two seesaw types [*)].

Soon after the observations of Refs. 17) and 18), it was argued that the MSSU(5) with a seesaw of type I, precisely because it induces sQFVs in the right-right down-squark sector, could accommodate values for B_s-$\bar{B}_s$ mixing (at that time unmeasured) distinguishable from the typical SM ones,[27)] without upsetting the observed agreement between experimental results and SM predictions for other FCNC processes. In contrast, predictions for this mixing in the MSSM with universal boundary conditions of the soft parameters (usually assumed at $M_{\rm GUT}$) tend to deviate less from those obtained in the SM.

The now existing measurement of ΔM_s, unfortunately, turns out to be inconclusive for searches of new physics. It does not show disagreement with the SM, but, in spite of its outstanding precision,[28)] it cannot exclude new-physics contributions either, because of the $\sim 30\%$ error that plagues the SM calculation.[29)] Recently, a new upsurge of interest in the MSSU(5) with RHNs was induced by the fact that the global fit for the phase of this mixing made by the UTfit collaboration, based on experimental data from D0[30)] and CDF,[31)] seemed to be 3.7σ away from the SM value.[32)] A subsequent reanalysis,[33)] as well as a fit from the HFAG,[34)] shows now a more modest 2.9 or 2.5σ deviation. Possible deviations of this phase from the SM value would be rather interesting, because its calculation is much less uncertain than that for ΔM_s.[35)] Surprisingly, however, the claim that the MSSU(5) model with a seesaw of type I could have some difficulty in explaining an anomaly at the 3.7σ level,[23)] when this seemed to be the correct value, did not originate an exploration of flavour and CP violation in the MSSU(5) model with the other two seesaw types. It would be important to perform such studies, since the relations between sLFVs and sQFVs, as well as the impact of GUT/seesaw phases may turn out to be largely different in the three cases.

In view of new and more extensive phenomenological studies, it is time to assess the state-of-the-art of the theoretical treatment of the MSSU(5) model. As is well known, this model is not realistic as it predicts a too rapid proton decay[36)] and wrong relations between the down-quark and charged-lepton masses.[37)] Suitable extensions

[*)] Exceptions are the papers of Refs. 24)–26), in which specific boundary conditions for the soft masses, universal[24)] or obtained from a combined gauge- and Yukawa-mediation mechanism of SUSY breaking,[25),26)] are imposed at $M_{\rm GUT}$.

of the field content,[37)–39)] or the inclusion of nonrenormalizable operators (NROs) suppressed by $1/M_{\rm cut}$, are believed to be sufficient to cure these problems. Since the former extensions are more obtrusive, as they bring larger modifications to the model, we opt here for the second possibility.

1.1. *Existing studies of NROs in the MSSU(5)*

The consequences that NROs may have for sFVs were first discussed in Refs. 40) in the context of the SUSY SO(10) model. They were also studied in Ref. 41) in the SUSY SU(5) model with vanishing neutrino masses, and were then neglected until the work of Ref. 19), where the seesaw of type I was implemented to obtain massive neutrinos. In Ref. 19), only one NRO of dimension five in the Yukawa sector was included (the minimal number sufficient to obtain a suitable fermion spectrum) [*)], nevertheless, the correlations between sLFVs and sQFVs induced by the seesaw Yukawa couplings were shown to be sizably altered.

The direct effect that NROs have through RGEs is at most of $\mathcal{O}(10^{-4})$, for the usual hierarchy between $M_{\rm cut}$ and $M_{\rm GUT}$. Although small, it may not be neglected if the seesaw couplings happen to be somewhat smaller than $\mathcal{O}(1)$. For this reason, the authors of Ref. 19) aimed at collecting all contributions of $\mathcal{O}(10^{-4})$. The largest effect that NROs have, however, comes from the arbitrariness they introduce in the choice of the flavour rotations of the SM fields to be embedded in the SU(5) matter multiplets. This is expressed by the appearance of unitary matrices of mismatch in the diagonalization (hereafter mismatch matrices) of various Yukawa couplings, in addition to the RGE-evolved CKM and MNS ones. There are two such matrices in the analysis of Ref. 19).

Since the coefficient of the NRO of Ref. 19) is tuned to provide corrections of O(1) to the Yukawa couplings of first and second generation fermions, it is somewhat obvious that these two mismatch matrices modify the pattern of sFVs in the first-second generation down-squark sector. What is perhaps less obvious is the fact that they can affect also, in a sizable way, the pattern of sFVs in which the third-generation down squark is involved.[22)] Unfortunately, the authors of Ref. 19) failed to emphasize this point, thereby implicitly substantiating the perception that the predictions for FCNC processes involving the bottom quark remain unaffected by the inclusion of NROs.

The authors of Refs. 40) and 41), included in their analyses all possible NROs in the Yukawa sector of the superpotential. To deal with such a complex situation, they made use of the picture of effective couplings. For each renormalizable operator, these collect at the tree level the coupling of the operator itself and the corrections contributed by different NROs with superheavy fields replaced by their vacuum-expectation values (*vevs*). This picture was assumed by these authors to be valid also at the quantum level.

No effort was made by any of the three groups of authors to verify whether the proton-decay rate can be suppressed in their scenarios. In the case of Ref. 19),

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[*)] After Ref. 19), the only other papers including NROs in the same context are those in Ref. 22), which follow quite closely the treatment of Ref. 19), and the paper in Ref. 42), which deals with a SUSY SO(10) model.

this seems hardly to be the case, given the presence of only one NRO. Enough SU(5)-breaking effects are induced by NROs in the analyses of Refs. 40) and 41). These can disentangle[43)–45)] the Yukawa couplings giving rise to the fermion masses from the Yukawa couplings contributing to the coefficients of the effective operators responsible for proton decay. The corresponding rate can then be tuned to be smaller than the existing experimental limits,[46)] also for colored Higgs fields with mass of $\mathcal{O}(M_{\rm GUT})$. Nevertheless, no mention of this issue is made in these papers, and no awareness appears in them of the fact that this tuning can have a substantial feedback into the sFVs problem.

1.2. *This paper*

Although closer in method to the treatment of NROs proposed in Ref. 40), this paper is the first step towards a generalization of Ref. 19) for all three seesaw types, with no restrictions on the type and number of NROs to be included. The resulting models are often dubbed in this analysis nrMSSU(5) models, to distinguish them from the MSSU(5) model without NROs. Special attention is paid to the possible shortcomings of the nrMSSU(5) models, in particular the loss of predictability, and the breaking of minimality in their Kähler potentials.

As in Ref. 19) we aim at collecting all the leading RGEs-induced effects due to NROs.

We start by reviewing the MSSU(5) model in Sec. 2.1 and the three types of the seesaw mechanism in Sec. 2.3. We study in Sec. 2.2 the vacuum structure, of the MSSU(5) model. We give analytic expressions for the scalar and auxiliary *vevs* of the Higgs field in the adjoint representation, 24_H. To the best of our knowledge, the expression of the auxiliary *vev* is given here explicitly for the first time. Barring the use of some extreme values for the parameters in the Higgs sector of the model, the *vevs* obtained in this section are, to a good approximation, also those for the nrMSSU(5) models.

Still in Sec. 2.2 we show how the tunings needed in the MSSU(5) model to induce light Higgs boson masses are stable under radiative corrections, irrespective of the type of soft parameters assumed at $M_{\rm cut}$. The proof is already given in a compact form in Ref. 47). Ours, is more direct and explicit, and uses the RGEs for the leading components of the *vevs* of the field 24_H in the expansion in powers of $(\widetilde{m}/M_{\rm GUT})$. We derive them in this section in terms of the original parameters of the model, and in a practically model-independent way in Appendix C. Thus, also this proof remains valid when NROs are included.

In Sec. 3 we survey in full generality the different NROs that can appear in the SUSY-conserving sector of the nrMSSU(5) models, and we list explicitly the various effective couplings induced. In Sec. 4 we show that, in general, the number of mismatch matrices, and therefore the amount of arbitrariness, introduced in the Yukawa sector, is considerably larger than in Ref. 19). In spite of this, only one of these matrices affects sizably the pattern of the seesaw-induced sFVs in the down-squark sector, as in the simpler case of Ref. 19).

We do not attempt an actual calculation of the proton-decay rate in this paper, limiting ourselves to recall old[43)–45)] and new[48)] possibilities to suppress it through

NROs. Nevertheless, in Sec. 5 we illustrate the consequences for sFVs of the ansätze used in Ref. 45) for its suppression. These correspond to specific choices of the effective Yukawa couplings of operators involving the colored Higgs triplets in the large parameter space opened up by the introduction of NROs. Interestingly, for these particular choices, the above mismatch matrices turn out to be well approximated by the unit matrix and their effects on the correlations between sQFVs and sLFVs to be negligible. We argue, however, that this is may not be a generic feature of these models and that a dedicated study should be devoted to this problem.

We survey in Sec. 6 the NROs that can appear in the SUSY-breaking part of the superpotential and Kähler potential. We give emphasis to the explicit expressions of the effective trilinear couplings and effective soft masses squared, to which also the auxiliary *vev* F_{24} contributes, a point which was missed in Ref. 19), and presumably also in Refs. 40) and 41). In reality, this *vev* has nontrivial consequences for the correct determination of the boundary conditions for effective soft couplings and for their RGEs.

We show in Sec. 7 that assuming boundary conditions for the original couplings of the nrMSSU(5) models consistent with a flavour- and field-independent mechanism of mediation of SUSY breaking, which induces universal boundary conditions in models without NROs, is not sufficient to avoid tree-level sFVs for effective couplings. After some guesses made in this section, we show in a systematic way in Sec. 8 how to avoid these tree-level sFVs by restricting further the type of couplings that the mediator of SUSY breaking can have to the various operators of the Kähler potential and the superpotential. Thanks to these restrictions, it is possible to recover for the effective soft coupling at $M_{\rm cut}$ the four-parameter description in terms of $M_{1/2}$, $\widetilde{m}_0$, A_0, B_0, which may be obtained also in the flat limit of minimal supergravity.[49)]

We are then faced with the problem of whether it is possible to upgrade this picture of effective couplings to the quantum level. In Sec. 10.1, we examine the possible loop diagrams that break this picture and we determine the level of accuracy in the calculation of sFVs for which this upgrade is doable. We show, in a sketchy way in Sec. 10.2 and a detailed one in Appendix D, that, within this level of accuracy, the RGEs for effective couplings are as those for renormalizable couplings, even above the GUT scale where the superheavy degrees of freedom are still active. This statement has already been claimed in Ref. 40), and then used also in Ref. 41), assuming that it is possible to neglect the wave function renormalization of the adjoint Higgs field. In reality, within our target accuracy, this approximation is not valid in the case of the effective trilinear couplings. Nevertheless, making use of the evolution equation of the scalar and auxiliary *vevs* derived in Sec. 2, we show that the formal equality of the RGEs for effective couplings and renormalizable couplings holds exactly, still within our target accuracy. This discussion is supplemented by complete lists of RGEs, given in Appendix E for all ranges of energies from $M_{\rm cut}$ to $M_{\rm weak}$, for each of the three seesaw types, correcting mistakes in some of the equations reported in the existing literature. The RGEs for effective couplings, as well as the RGEs for the seesaw of type II and III above $M_{\rm GUT}$ appear here for the first time.

Finally, in Sec. 11 we give approximated analytic expressions for low-energy

off-diagonal terms in the squark and slepton mass matrices induced by the seesaw couplings, for vanishing and nonvanishing NROs. Those for vanishing NROs are still predictions for the flawed MSSU(5) model, which are however missing in the existing literature. We show explicitly how NROs can modify the seesaw-induced transitions between squarks of first and second generation, as well as first and third and second and third generation. We summarize our analysis in Sec. 12.

1.3. *Notation issues*

We do not distinguish in this paper fundamental and antifundamental indices as upper and lower ones, as it is usually done. We give, however, in Appendix A explicit multiplication rules for each term of the superpotential and Kähler potential, clarifying our notation. In the same Appendix we give also explanatory details for many of the equations appearing in the text. In particular, the SU(5) generators T_i listed there are assumed to be acting on the antifundamental representation. Thus, they act as $-T_i^T 5$ on the fundamental representation 5. Throughout this paper, we use the same symbol for Higgs superfields and their scalar components. An exception is made only for the field 24_H in Sec. 2.2 and Appendix D. Finally, a dot on a parameter $P(Q)$ denotes the partial derivative $(16\pi^2)\partial P/\partial \ln(Q/Q_0)$, with Q_0 an arbitrary reference scale.

§2. The MSSU(5) model with seesaw

In the MSSU(5) model,[50] the supersymmetric version of the Georgi-Glashow SU(5) model,[51] the superpotential can be split as:

$$W^{\mathrm{MSSU(5)}} = W_{\mathrm{M}}^{\mathrm{MSSU(5)}} + W_{\mathrm{H}}^{\mathrm{MSSU(5)}}, \tag{2·1}$$

where $W_{\mathrm{M}}^{\mathrm{MSSU(5)}}$ contains all matter interactions:

$$W_{\mathrm{M}}^{\mathrm{MSSU(5)}} = \sqrt{2}\,\bar{5}_M Y^5\,10_M \bar{5}_H - \frac{1}{4} 10_M Y^{10}\,10_M 5_H, \tag{2·2}$$

and $W_{\mathrm{H}}^{\mathrm{MSSU(5)}}$ all Higgs interactions:

$$W_{\mathrm{H}}^{\mathrm{MSSU(5)}} = M_5 5_H \bar{5}_H + \lambda_5 5_H 24_H \bar{5}_H + \frac{1}{2} M_{24}(24_H)^2 + \frac{1}{6}\lambda_{24}(24_H)^3. \tag{2·3}$$

Here Y^{10} is a symmetric matrix, whereas Y^5 is a generic one. As is well known, the two irreducible representations $\bar{5}_M$ and 10_M collect, generation by generation, all matter superfields of the MSSM: $10_M = \{Q, U^c, E^c\}$ and $\bar{5}_M = \{D^c, L\}$. The two Higgs fields 5_H and $\bar{5}_H$ contain the two weak Higgs doublets of the MSSM, H_u and H_d, and the two colour triplets, H_U^C and H_D^C: $5_H = \{H_U^C, H_u\}$ and $\bar{5}_H = \{H_D^C, H_d\}$. The explicit form of the Higgs field 24_H is given in Appendix A, where also the SM decomposition of most superpotential terms can be found. It is sufficient here to say that its components are: G_H and W_H, respectively in the adjoint representations of SU(3) and SU(2); X_H, an SU(2) doublet and SU(3) antitriplet; $\bar{X}_H$, an SU(2) antidoublet and SU(3) triplet; B_H, a SM singlet.

In this form, the MSSU(5) model predicts vanishing neutrino masses. The extension usually made to obviate this problem consists in the introduction of a heavy seesaw sector. Depending on the particle content of this sector, three realizations of the seesaw mechanism are possible, denoted as seesaw of type I, II, and III, which will be discussed in Sec. 2.3. We also postpone a discussion of the vacuum structure of the model to Sec. 2.2, whereas we concentrate in this coming section to the quark and lepton interactions.

2.1. *matter sector*

After performing some rotations in flavour space discussed in Appendix B, it is possible to recast $W_{\rm M}^{\rm MSSU(5)}$ in the form:

$$W_{\rm M}^{\rm MSSU(5)} = \sqrt{2}\,\bar{5}_M \widehat{Y}^5\, 10_M \bar{5}_H - \frac{1}{4} 10_M \left(K_{10}^T P_{10}\, \widehat{Y}^{10}\, K_{10}\right) 10_M 5_H, \tag{2·4}$$

which shows explicitly the physical parameter of this superpotential term: K_{10} and P_{10}, which like all K- and P-matrices throughout this paper, are respectively a unitary matrix with three mixing angles and one phase, and a diagonal phase matrix with determinant equal to one, and the real and positive entries in the diagonal matrices of Yukawa couplings, $\widehat{Y}^5$ and $\widehat{Y}^{10}$.

By decomposing the two terms in Eq. (2·4) in SM representations, we can split $W_{\rm M}^{\rm MSSU(5)}$ as:

$$W_{\rm M}^{\rm MSSU(5)} = W'^{\,\rm MSSM} + W_{\rm matt-H^C}^{\rm MSSU(5)}, \tag{2·5}$$

with $W'^{\,\rm MSSM}$ containing only MSSM fields, $W_{\rm matt-H^C}^{\rm MSSU(5)}$ collecting all the Yukawa interactions involving Higgs triplets. The following identification of the MSSM fields Q, D^c, U^c, L and E^c among the components of the MSSU(5) fields $\bar{5}_M$ and 10_M:

$$\bar{5}_M \to \{D^c, e^{-i\phi_l} P_l^\dagger L\}, \qquad 10_M \to \{Q, K_{10}^\dagger P_{10}^\dagger U^c, e^{i\phi_l} P_l E^c\}, \tag{2·6}$$

removes the so-called GUT phases P_{10} from the MSSM-like part, and reduces the up-quark Yukawa coupling to be nonsymmetric. After the renaming:

$$\widehat{Y}^{10} \to \widehat{Y}_U, \qquad \widehat{Y}^5 \to \widehat{Y}_D, \qquad K_{10} \to K_{\rm CKM}, \tag{2·7}$$

where $\widehat{Y}_D^5$, $\widehat{Y}_U^{10}$ and $K_{\rm CKM}$ are, respectively, the diagonal matrices of down- and up-quark Yukawa couplings, ${\rm diag}(y_d, y_s, y_b)$, ${\rm diag}(y_u, y_c, y_t)$, and the CKM matrix, all at $M_{\rm GUT}$, the MSSM-like becomes:

$$W'^{\,\rm MSSM} = U^c \left(\widehat{Y}_U K_{\rm CKM}\right) Q H_u - D^c\, \widehat{Y}_D\, Q H_d - E^c\, \widehat{Y}_D\, L H_d, \tag{2·8}$$

with the down-quark diagonal Yukawa matrix also in the leptonic term. In its place the MSSM superpotential has a different diagonal matrix $\widehat{Y}_E$, with three additional independent parameters, $\widehat{Y}_E \equiv {\rm diag}(y_e, y_\mu, y_\tau)$.

Notice that the above identification is not unique, as nonunique are also the parametrizations of Y^5 and Y^{10} in terms of the physical parameters of $W_{\rm M}^{\rm MSSU(5)}$, and that of the MSSM Yukawa matrices. (See Appendix B.) The appearance of the

three phases $e^{i\phi_l}P_l$ shows the arbitrariness that still remains in choosing the SU(2) doublets and singlet lepton fields, L and E^c. This will be used to remove three phases in the seesaw sector.

With this identification and the form of $W'^{\rm MSSM}$ in Eq. (2·8), $W^{\rm MSSU(5)}_{\rm matt-H^C}$, looks as:

$$W^{\rm MSSU(5)}_{\rm matt-H^C} = \frac{1}{2}Q\left(K^T_{\rm CKM}\widehat{Y}_U P_{10} K_{\rm CKM}\right) QH^C_U + e^{+i\phi_l}\,U^c\left(\widehat{Y}_U K_{\rm CKM} P_l\right) E^c H^C_U$$
$$-e^{-i\phi_l}\,L\left(P^\dagger_l \widehat{Y}_D\right) QH^C_D - D^c\left(\widehat{Y}_D K^\dagger_{\rm CKM} P^\dagger_{10}\right) U^c H^C_D. \qquad (2\cdot 9)$$

It contains all the phases P_{10} and $e^{i\phi_l}P_l$ rotated away from $W'^{\rm MSSM}$. Consistently, once the Higgs triplets are integrated out, their dependence disappear from the superpotential below $M_{\rm GUT}$. A trace of these phases, however, remains in the modification that the soft SUSY-breaking parameters undergo through renormalization from $M_{\rm cut}$ down to $M_{\rm GUT}$. They remain also in higher-dimension operators generated by integrating out of the colored Higgs triplets, such as the proton-decay operators discussed in Sec. 5.

The term $W'^{\rm MSSM}$ still differs from the conventional MSSM superpotential as it lacks the bilinear terms in the two Higgs doublets, with massive coupling μ. Up to soft SUSY-breaking terms, this parameter and the dynamics leading to the breaking of SU(5) are determined by $W^{\rm MSSU(5)}_{\rm H}$.

2.2. *Higgs sector and SU(5)-breaking vacuum.*

We assume that the field 24_H acquires a non-vanishing *vev* of $\mathcal{O}(M_{\rm GUT})$ only in the hypercharge direction, $\langle 24^{24}_H\rangle$, since 24^{24}_H is the only SM singlet among its components *). Elsewhere, this field is denoted by the symbol B_H, which indicates both, the complete superfield and its scalar component, like all symbols used for Higgs fields in this paper. In this section and in Appendix D, where the double use of the same symbol may generate some confusion, we reserve the symbol 24^{24}_H for the superfield, whereas B_H and F_{B_H} are used for its scalar and auxiliary components.

The *vev* 24^{24}_H is decomposed as:

$$\langle 24^{24}_H\rangle = \langle B_H\rangle + \theta^2\langle F_{B_H}\rangle, \qquad (2\cdot 10)$$

and the part of the superpotential relevant for its determination is a subset of $W^{\rm MSSU(5)}_H$ in Eq. (2·3):

$$W_{24^{24}_H} = \frac{1}{2}M_{24}\left(24^{24}_H\right)^2 - \frac{1}{6\sqrt{30}}\lambda_{24}\left(24^{24}_H\right)^3. \qquad (2\cdot 11)$$

*) The neutral component of W_H in 24_H, colorless, could also acquire a *vev* once the electroweak symmetry is broken. A dimensional analysis shows that its scalar component, with mass squared of $\mathcal{O}(M^2_{\rm GUT})$, can induce a tadpole term at most of $\mathcal{O}(\widetilde{m}^2 M_{\rm GUT})$, and therefore a *vev* at most of $\mathcal{O}(\widetilde{m}^2/M_{\rm GUT})$, *i.e.* completely negligible. We set to zero the *vevs* of 5_H and $\bar{5}_H$, which are also negligible and completely irrelevant for this analysis. Since all the components of 24_H have masses of $\mathcal{O}(M^2_{\rm GUT})$, the vacuum $\langle 24^{24}_H\rangle$ is bound to remain a vacuum, at least locally, even when the SUSY-breaking terms are included, which have the only the effect of producing tiny shifts in its value.

For vanishing soft SUSY-breaking terms, the scalar potential determining the vacuum is simply given by $|F_{B_H}|^2$, with:

$$F^*_{B_H} = \frac{\partial W(B_H)}{\partial B_H} = M_{24} B_H - \frac{1}{2\sqrt{30}} \lambda_{24} B_H^2. \tag{2·12}$$

For $F_{B_H} = 0$, a nonvanishing *vev* for B_H is generated:

$$\langle B_H \rangle = v_{24} \equiv 2\sqrt{30}\, \frac{M_{24}}{\lambda_{24}}, \qquad (\langle F_{B_H} \rangle = 0)\,. \tag{2·13}$$

Notice that v_{24} and $\langle B_H \rangle$ coincide only in the limit of exact SUSY, but they differ for nonvanishing SUSY-breaking terms. The *vev* v_{24} splits the masses of the doublet and triplet components of the 5_H and $\bar{5}_H$ Higgs fields, which we call μ_2 and μ_3:

$$\mu_2 = M_5 - \frac{1}{2}\sqrt{\frac{6}{5}} \lambda_5 v_{24} = M_5 - 6\lambda_5\, \frac{M_{24}}{\lambda_{24}}, \tag{2·14}$$

$$\mu_3 = M_5 + \frac{1}{3}\sqrt{\frac{6}{5}} \lambda_5 v_{24} = M_5 + 4\lambda_5\, \frac{M_{24}}{\lambda_{24}} = \mu_2 + 10\lambda_5\, \frac{M_{24}}{\lambda_{24}}. \tag{2·15}$$

It is clear that M_5 and M_{24}, both of $\mathcal{O}(M_{\rm GUT})$, must be fine tuned to render μ_2 of $\mathcal{O}(M_{\rm weak})$, whereas μ_3 is naturally of $\mathcal{O}(M_{\rm GUT})$, *i.e.* heavy enough not to disturb the success of the gauge coupling unification. This fine-tuning problem, or doublet-triplet splitting problem, is generic of GUT models and has spurred many proposals for its solution.[38),53)–58)] The known remedies in the case of SU(5) in four dimensional space-time[38)] require a departure from the assumption of minimality encoded in $W_{\rm H}^{\rm MSSU(5)}$. Here, we keep this assumption and we agree to tolerate the severe fine tuning of Eq. (2·14), which is stable under radiative corrections thanks to the nonrenormalization theorem for superpotentials. The soft SUSY-breaking scalar potential is not equally protected and, in general, we expect that the fine-tuning to be performed also in this sector is potentially destabilized by radiative corrections. This turns out not to be the case, and both tunings are stable.[47)]

To show this, we start by giving the form of the soft SUSY-breaking scalar potential:

$$\widetilde{V}^{\rm MSSU(5)} = \widetilde{V}^{\rm MSSU(5)}_{\rm M} + \widetilde{V}^{\rm MSSU(5)}_{\rm H} + \widetilde{V}^{\rm MSSU(5)}_{\rm gaug}, \tag{2·16}$$

split into a matter part [*)], a purely Higgs-boson part, and a gaugino part:

$$\begin{aligned}
\widetilde{V}^{\rm MSSU(5)}_{\rm M} &= \left\{ -\frac{1}{4} \widetilde{10}_M A^{10} \widetilde{10}_M 5_H + \sqrt{2}\, \widetilde{\bar{5}}_M A^5 \widetilde{10}_M \bar{5}_H + {\rm H.c.} \right\} + \widetilde{10}^*_M \widetilde{m}^2_{10_M} \widetilde{10}_M \\
&\qquad + \widetilde{\bar{5}}_M \widetilde{m}^2_{\bar{5}_M} \widetilde{\bar{5}}^*_M, \\
\widetilde{V}^{\rm MSSU(5)}_{\rm H} &= \left\{ B_5 5_H \bar{5}_H + A_{\lambda_5} 5_H 24_H \bar{5}_H + \frac{1}{2} B_{24} (24_H)^2 + \frac{1}{6} A_{\lambda_{24}} (24_H)^3 + {\rm H.c.} \right\} \\
&\quad + \widetilde{m}^2_{5_H} 5^*_H 5_H + \widetilde{m}^2_{\bar{5}_H} \bar{5}^*_H \bar{5}_H + \widetilde{m}^2_{24_H} 24^*_H 24_H, \\
\widetilde{V}^{\rm MSSU(5)}_{\rm gaug} &= \frac{1}{2} M_G \widetilde{G}_5 \widetilde{G}_5.
\end{aligned} \tag{2·17}$$

*) Notice the different conventions for the soft masses of the fields 10_M and $\bar{5}_M$, discussed below Eq. (E·25) in Appendix E.

The inclusion of the soft SUSY-breaking terms shifts the scalar potential relevant for the determination of the MSSU(5) vacuum as follows:

$$V_{B_H} = |F_{B_H}|^2 + \widetilde{V}_{B_H}, \tag{2·18}$$

where $\widetilde{V}_{B_H}$ is a subset of the potential $\widetilde{V}_{\rm H}^{\rm MSSU(5)}$ in Eq. (2·17):

$$\widetilde{V}_{B_H} = \widetilde{m}^2_{24_H} B_H^* B_H + \left\{ \frac{1}{2} B_{24} B_H^2 - \frac{1}{6\sqrt{30}} A_{\lambda_{24}} B_H^3 + {\rm H.c.} \right\}. \tag{2·19}$$

Once the soft terms are introduced, in general, also the auxiliary component of 24_H^{24} acquires a *vev* and the value of the scalar-component *vev* gets modified.[16] The expressions for both these *vevs* can be obtained perturbatively, using $(\widetilde{m}/M_{\rm GUT})$ as expansion parameter, where $\widetilde{m}$ is a typical soft mass:

$$\begin{aligned}
\langle B_H \rangle &= v_{24} + \delta v_{24} + \delta^2 v_{24} + \mathcal{O}\left(\frac{\widetilde{m}^3}{M_{\rm GUT}^2}\right), \\
\langle F_{B_H} \rangle &= \ 0 \ + \ F_{24} \ + \ \delta F_{24} + \mathcal{O}\left(\frac{\widetilde{m}^3}{M_{\rm GUT}}\right).
\end{aligned} \tag{2·20}$$

Here, v_{24}, δv_{24}, and $\delta^2 v_{24}$ are respectively of $\mathcal{O}(M_{\rm GUT})$, $\mathcal{O}(\widetilde{m})$, and $\mathcal{O}(\widetilde{m}^2/M_{\rm GUT})$, whereas F_{24} and δF_{24} are of $\mathcal{O}(\widetilde{m} M_{\rm GUT})$ and $\mathcal{O}(\widetilde{m}^2)$. A straightforward calculation yields:

$$\begin{aligned}
\delta v_{24} &= -\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^* \frac{v_{24}^*}{M_{24}}, \\
F_{24} &= \left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right) v_{24}, \\
\delta F_{24} &= \left[\widetilde{m}^2_{24_H} + \frac{B_{24}}{M_{24}}\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*\right] \frac{v_{24}^*}{M_{24}}.
\end{aligned} \tag{2·21}$$

The MSSM μ and B parameters are then expressed in term of $\langle B_H \rangle$ and $\langle F_{B_H} \rangle$ [*]:

$$\begin{aligned}
\mu &= M_5 - \frac{1}{2}\sqrt{\frac{6}{5}} \lambda_5 \langle B_H \rangle \equiv \mu_2 + \delta\mu_2 + \mathcal{O}\left(\frac{\widetilde{m}^2}{M_{\rm GUT}}\right), \\
B &= B_5 - \frac{1}{2}\sqrt{\frac{6}{5}} \left(A_{\lambda_5} \langle B_H \rangle + \lambda_5 \langle F_{B_H} \rangle\right) \equiv B_2 + \delta B_2 + \mathcal{O}\left(\frac{\widetilde{m}^3}{M_{\rm GUT}}\right),
\end{aligned} \tag{2·22}$$

where μ_2 is already defined in Eq. (2·14), and B_2 is

$$B_2 = B_5 - \frac{1}{2}\sqrt{\frac{6}{5}} \left(A_{\lambda_5} v_{24} + \lambda_5 F_{24}\right) = \frac{B_5}{M_5}\mu_2 + 6\frac{\lambda_5}{\lambda_{24}} M_{24}\, \Delta, \tag{2·23}$$

[*] The importance of the correct matching of μ and B to the original GUT parameters is highlighted in Ref. 59) in the context of no-scale supergravity.

with $\varDelta$:

$$\varDelta \equiv \frac{B_5}{M_5} - \frac{A_{\lambda_5}}{\lambda_5} - \frac{B_{24}}{M_{24}} + \frac{A_{\lambda_{24}}}{\lambda_{24}}. \tag{2·24}$$

Finally, $\delta\mu_2$ and δB_2 are

$$\begin{aligned}
\delta\mu_2 &= -\frac{1}{2}\sqrt{\frac{6}{5}}\lambda_5\delta v_{24} = \; 6\frac{\lambda_5}{\lambda_{24}^*}\frac{M_{24}^*}{M_{24}}\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*, \\
\delta B_2 &= -\frac{1}{2}\sqrt{\frac{6}{5}}\left[A_{\lambda_5}\delta v_{24} + \lambda_5\delta F_{24}\right] \\
&\qquad\qquad = -6\frac{\lambda_5}{\lambda_{24}^*}\frac{M_{24}^*}{M_{24}}\left[\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_5}}{\lambda_5}\right) + \widetilde{m}^2_{24_H}\right].
\end{aligned} \tag{2·25}$$

The quantities to be fine tuned in these equations are μ_2, and B_2, and the tuning conditions to be imposed are:

$$\mu_2 = \mathcal{O}(\widetilde{m}), \qquad \varDelta = \mathcal{O}\left(\frac{\widetilde{m}^2}{M_{\rm GUT}}\right). \tag{2·26}$$

That both these tuning conditions are stable against radiative corrections, as claimed above, can be seen by making use of the RGEs listed in Appendix E.3 for the parameters of the Higgs sector entering the second equalities for μ_2 and $\varDelta$ in Eqs. (2·14) and (2·24).

The calculation becomes particularly simple in terms of the RGE for v_{24}, which we give here together with that for F_{24}:

$$\begin{aligned}
\dot{v}_{24} &= -\gamma_{24}\, v_{24}, \\
\dot{F}_{24} &= -\gamma_{24} F_{24} - \tilde{\gamma}_{24}\, v_{24}.
\end{aligned} \tag{2·27}$$

These evolution equations, expressed in terms of the anomalous dimension of the field 24_H, γ_{24}, and its soft counterpart $\tilde{\gamma}_{24}$, are obtained in a very general way in Appendix C. Both *vevs* exist above $M_{\rm GUT}$. The *vev* v_{24}, in particular, determined by superpotential parameters, is very different, for example, from the MSSM *vevs* v_u and v_d, determined by Kähler potential parameters and whose existence is not guaranteed at all scales. The RGE for F_{24} shows that a nonvanishing value for this *vev* is generated through radiative corrections, even when starting with a vanishing one at some scale. Since it will be of use later on, we define here also the quantity f_{24}, *i.e.* the ratio of the two *vevs*:

$$f_{24} \equiv \frac{F_{24}}{v_{24}} = \left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right), \tag{2·28}$$

with the simple evolution equation:

$$\dot{f}_{24} = -\tilde{\gamma}_{24}. \tag{2·29}$$

The RGEs for δv_{24}, δF_{24} and $\delta^2 v_{24}$, are very different as these quantities depend on the Kähler potential (see Appendix C), which gets vertex corrections at the loop level.

The first RGE in Eq. (2·27) has as a consequence the fact that $\lambda_5 v_{24}$ evolves as M_5, and so does also μ_2:

$$\dot{\mu}_2 = \left(\gamma_{5_H} + \gamma_{\bar{5}_H}\right) \mu_2. \tag{2·30}$$

Since the quantity Δ turns out to be scale invariant:

$$\dot{\Delta} = 0, \tag{2·31}$$

also B_2 evolves like B_5:

$$\dot{B}_2 = \left(\gamma_{5_H} + \gamma_{\bar{5}_H}\right) B_2 + \left(\tilde{\gamma}_{5_H} + \tilde{\gamma}_{\bar{5}_H}\right) \mu_2. \tag{2·32}$$

This is sufficient to prove that the tuning for μ_2 and B_2 is not destabilized by quantum corrections. Notice that no specific ansatz was made for the boundary conditions assigned to the trilinear couplings A_{λ_5} and $A_{\lambda_{24}}$, and the bilinear couplings B_5 and B_{24}, except that they are of $\mathcal{O}(\widetilde{m})$ and $\mathcal{O}(\widetilde{m} M_{\rm GUT})$, respectively.

On the other hand, the specific values chosen for μ_2 and Δ in Eq. (2·26) can affect the values of the boundary conditions for the MSSM parameters μ and B. We take as example minimal supergravity, in which at $M_{\rm cut}$, the soft masses for all scalar fields are $\widetilde{m}_0$, all ratios of trilinear couplings over the corresponding Yukawa couplings are $(A_{\lambda_i}/\lambda_i)(M_{\rm cut}) = A_0$, and all ratios of heavy bilinear parameters over the corresponding superpotential masses are $(B_i/M_i)(M_{\rm cut}) = B_0$, with

$$B_0 = A_0 - \widetilde{m}_0. \tag{2·33}$$

In this context, at $M_{\rm cut}$, it is $\Delta = 0$, and if the value of $\mu_2(M_{\rm cut}) = 0$ is imposed, we obtain for the boundary condition of the MSSM μ parameter: $\mu(M_{\rm cut}) = \delta\mu_2(M_{\rm cut})$, with

$$\delta\mu_2(M_{\rm cut}) = -6\widetilde{m}_0 \left[\left(\frac{\lambda_5}{\lambda_{24}^*}\right)\left(\frac{M_{24}^*}{M_{24}}\right)\right](M_{\rm cut}), \tag{2·34}$$

and for the boundary condition of the MSSM B parameter:[16)]

$$B(M_{\rm cut}) = 2\widetilde{m}_0 \delta\mu_2(M_{\rm cut}). \tag{2·35}$$

In contrast, if $\mu_2(M_{\rm cut})$ (and therefore $\mu(M_{\rm cut})$) is a nonvanishing arbitrary quantity of $\mathcal{O}(\widetilde{m})$, the relation between the boundary conditions of B and μ is also arbitrary:

$$B(M_{\rm cut}) = \left[B_0 \mu_2(M_{\rm cut}) + 2\widetilde{m}_0 \delta\mu_2(M_{\rm cut})\right], \tag{2·36}$$

with B_0 in Eq. (2·33) and $\delta\mu_2(M_{\rm cut})$ in Eq. (2·34).

For completeness we give here also the tree-level mass of the triplet Higgs fields including the small correction $\delta\mu_3$, of $\mathcal{O}(\widetilde{m})$, due to the term δv_{24} of the *vev* $\langle B_H \rangle$:

$$M_{H^C} \equiv \mu_3 + \delta\mu_3 = \mu + 10\lambda_5 \frac{M_{24}}{\lambda_{24}} - 10 \frac{\lambda_5}{\lambda_{24}} \frac{M_{24}^*}{M_{24}} \left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*. \tag{2·37}$$

We also collect together the various pieces of the parameters μ and B:

$$\mu = \left[M_5 - 6\lambda_5 \frac{M_{24}}{\lambda_{24}}\right] + 6 \frac{\lambda_5}{\lambda_{24}^*} \frac{M_{24}^*}{M_{24}} \left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*,$$

$$B = \left[B_5 - 6\frac{\lambda_5}{\lambda_{24}}M_{24}\left(\frac{A_{\lambda_5}}{\lambda_5} + \frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)\right] - 6\frac{\lambda_5}{\lambda_{24}^*}\frac{M_{24}^*}{M_{24}}\left[\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_{24}}}{\lambda_{24}}\right)^*\left(\frac{B_{24}}{M_{24}} - \frac{A_{\lambda_5}}{\lambda_5}\right) + \widetilde{m}^2_{B_H}\right], \quad (2{\cdot}38)$$

where the first square brackets in the two equations are μ_2 and B_2, respectively. Notice that, differently from μ_2, the quantities $\delta\mu_2$ and $\delta\mu_3$ do not evolve like all the superpotential dimensionful parameters. In a similar way, δB_2 does not evolve like a typical bilinear soft parameter, whereas B_2 does. This is because their expressions contain δv_{24} and δF_{24}. Thus, above $M_{\rm GUT}$, also the MSSM μ and B parameters, as well as the small correction to M_{H^C} of $\mathcal{O}(\widetilde{m})$, have nonconventional RGEs.

The derivation of $\langle B_H\rangle$ and $\langle F_{B_H}\rangle$ in this section allows for independent phases of M_{24} and λ_{24}. It is possible, however, to redefine the field 24_H in such a way to align them. That is to say, without any loss of generality, it is possible to take v_{24} to be real, which is what will be assumed hereafter. In contrast, the phases of the shifts δv_{24}, F_{24}, and δF_{24} depend on those of the bilinear and trilinear terms in $\widetilde{V}_{\rm H}^{\rm MSSU(5)}$.

2.3. *Seesaw sector*

The seesaw mechanism is a mechanism that generates the effective dimension-five operator for Majorana neutrino masses:

$$W_\nu = -\frac{1}{2}LH_u\kappa LH_u, \quad (2{\cdot}39)$$

where the symmetric matrix κ is a dimensionful parameter of $O(M_{\rm ssw}^{-1})$. This operator is obtained by integrating out the heavy seesaw degrees of freedom at the scale $M_{\rm ssw}$. The mechanism is depicted in Fig. 1, where a solid (broken) line indicates a fermion (boson) or, in a supersymmetric context, a superfield with a Z_2 odd (even) R-parity. At the tree level, there are only two diagrams that can give rise to the above effective operator, one mediated by a solid line and one by a broken line.

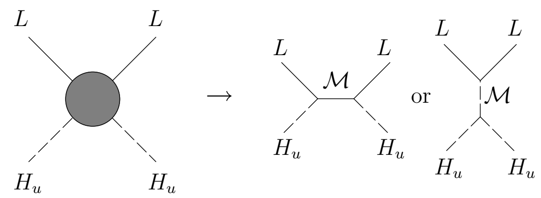

Fig. 1. The seesaw mechanism.

Taking into account the gauge structure, it may seem at first glance that in both cases, the inner line that represents the mediator $\mathcal{M}$ can be a singlet or triplet of SU(2). In reality, the possibility of the singlet scalar mediator $\mathcal{M}$ is forbidden by the multiplication rule of SU(2): $2\times 2 = 1_A + 3_S$, where the indices A and S indicate an antisymmetric and a symmetric product. Thus, there are only three types of seesaw mechanism, with mediators:

singlet fermions	:	type I,
a triplet scalar	:	type II,
triplet fermions	:	type III,

i.e. the RHNs N^c, a triplet Higgs boson T, and matter triplets W_M, respectively. They interact with the lepton doublets through the large Yukawa couplings Y_ν^i ($i = \mathrm{I}, \mathrm{II}, \mathrm{III}$):

$$N^c Y_\nu^{\mathrm{I}} L H_u, \qquad \frac{1}{\sqrt{2}} L Y_\nu^{\mathrm{II}} T L, \qquad H_u W_M Y_\nu^{\mathrm{III}} L. \tag{2·40}$$

With a minimal completion, we obtain the seesaw superpotentials *):

$$\begin{aligned} W_{\mathrm{ssw\,I}} &= N^c Y_\nu^{\mathrm{I}} L H_u + \frac{1}{2} N^c M_N N^c, \\ W'_{\mathrm{ssw\,II}} &= \frac{1}{\sqrt{2}} L Y_\nu^{\mathrm{II}} T L + \frac{1}{\sqrt{2}} \lambda_{\bar{T}} H_u \bar{T} H_u + M_T T \bar{T}, \\ W'_{\mathrm{ssw\,III}} &= H_u W_M Y_\nu^{\mathrm{III}} L + \frac{1}{2} W_M M_{W_M} W_M, \end{aligned} \tag{2·41}$$

where of the three superheavy masses M_N, M_T, and M_{W_M}, M_N and M_{W_M}, are two 3×3 matrices, M_T, just a number. Following Appendix B, it is easy to see that Y_ν^{I} and Y_ν^{III} are complex matrices with fifteen parameters, whereas Y_ν^{II} is a nine-parameter symmetric complex matrix.

By integrating out the mediators in the superpotentials of Eq. (2·41), and replacing H_u by its *vev* v_u, we obtain relations between the neutrino mass matrix, κv_u^2, and the seesaw Yukawa couplings:

$$m_\nu = \kappa v_u^2 = \begin{cases} \left(Y_\nu^{\mathrm{I}}\right)^T \dfrac{1}{M_N} \left(Y_\nu^{\mathrm{I}}\right) v_u^2 & \text{for type I,} \\ Y_\nu^{\mathrm{II}} \dfrac{1}{M_T} \lambda_{\bar{T}} v_u^2 & \text{for type II,} \\ \dfrac{1}{2} \left(Y_\nu^{\mathrm{III}}\right)^T \dfrac{1}{M_{W_M}} \left(Y_\nu^{\mathrm{III}}\right) v_u^2 & \text{for type III.} \end{cases} \tag{2·42}$$

(See Appendix A for the normalization of the fields W_M, which are in the adjoint representation of SU(2).)

The neutrino mass matrix is usually expressed as:

$$m_\nu = V_{\mathrm{MNS}}^* \, \widehat{m}_\nu V_{\mathrm{MNS}}^\dagger. \tag{2·43}$$

In the basis in which the Yukawa coupling for the charged leptons is diagonal, the MNS matrix V_{MNS}, a unitary matrix with three mixings and three phases, is the diagonalization matrix of m_ν. It is often parametrized as: $V_{\mathrm{MNS}}^* = K_{\mathrm{MNS}} P_{\mathrm{MNS}}$, where K_{MNS} is a K_{CKM}-like matrix, with three mixing angles and one phase, P_{MNS} is a two-phase matrix, the so-called Majorana phases.

In the same basis, it is possible to invert the relations in Eq. (2·42), solve for the couplings Y_ν^i (i= I,II,III) in terms of m_ν, and therefore obtain the high-energy neutrino parameters in terms of those that can be tested at low energy, V_{MNS} and

*) We distinguish by a prime the second and third superpotential from $W_{\mathrm{ssw\,II}}$ and $W_{\mathrm{ssw\,III}}$ listed in Appendix E, which contain more operators and fields, not needed for the implementation of the seesaw mechanism in the MSSM context, but necessary for the completion of the seesaw sectors of type II and III in the MSSU(5) model.

$\widehat{m}_\nu$. In the case of the seesaw of type II, because the mediator has no flavour, the high-energy input in the neutrino mass matrix is just a number, *i.e.* the ratio $\lambda_{\bar{T}}/M_{\mathcal{M}}^{\rm II}$, and the flavour structure of $Y_\nu^{\rm II}$ is the same as that of the neutrino mass m_ν. Thus, the expression for $Y_\nu^{\rm II}$ is rather simple:

$$Y_\nu^{\rm II} = \frac{1}{v_u^2} V_{\rm MNS}^* \,\widehat{m}_\nu V_{\rm MNS}^\dagger \frac{M_T}{\lambda_{\bar{T}}}. \tag{2·44}$$

The corresponding ones for type I and III, in which the mediators carry flavour indices, are far more involved. The neutrino mass matrix, which depends on a large number of high-energy parameters, has a flavour structure different from those of $Y_\nu^{\rm I}$ and $Y_\nu^{\rm III}$. In a basis in which also the mediator masses are diagonal, these couplings can be expressed as:

$$\begin{aligned}
\left(Y_\nu^{\rm I}\right)^T &= \frac{1}{v_u} V_{\rm MNS}^* \sqrt{\widehat{m}_\nu}\, R \sqrt{\widehat{M}_N}, \\
\left(Y_\nu^{\rm III}\right)^T &= \frac{\sqrt{2}}{v_u} V_{\rm MNS}^* \sqrt{\widehat{m}_\nu}\, R \sqrt{\widehat{M}_{W_M}}.
\end{aligned} \tag{2·45}$$

Here, an arbitrary complex orthogonal matrix R,[60] parametrized by three (arbitrary) mixing angles and three phases, appears. Notice also that in these two cases, m_ν is quadratic in $Y_\nu^{\rm I}$ and $Y_\nu^{\rm III}$, whereas in the seesaw of type II is linear in $Y_\nu^{\rm II}$.

In SU(5), the multiplets respectively containing the mediators of the seesaw of type I, II, III are matter singlets, or RHNs N^c, a 15plet Higgs field, 15_H, and three adjoint matter fields, 24_M, distinguished from the Higgs field 24_H by the index M, for matter. The components of the fields 24_M are denoted as G_M, W_M, X_M, $\bar{X}_M$, and B_M, whereas in addition to the triplet Higgs T, 15_H contains also the fields S and Q_{15}. (See Appendix A.) The complete seesaw superpotentials are [*)]

$$\begin{aligned}
W_{\rm RHN} &= -N^c Y_N^{\rm I} \bar{5}_M 5_H + \frac{1}{2} N^c M_N N^c, \\
W_{\rm 15H} &= \frac{1}{\sqrt{2}} \bar{5}_M Y_N^{\rm II} 15_H \bar{5}_M + M_{15} 15_H \bar{15}_H \\
&\quad + \frac{1}{\sqrt{2}} \lambda_D \,\bar{5}_H 15_H \bar{5}_H + \frac{1}{\sqrt{2}} \lambda_U \,5_H \bar{15}_H 5_H + \lambda_{15} 15_H 24_H \bar{15}_H, \\
W_{\rm 24M} &= 5_H 24_M Y_N^{\rm III} \bar{5}_M + \frac{1}{2} 24_M M_{24_M} 24_M + \frac{1}{2} \sum_{x=S,A} \left(24_M Y_{24_M}^x 24_M\right)_x 24_H,
\end{aligned} \tag{2·46}$$

where the index x in the last term of the third of these equations refers to the symmetric and antisymmetric product of the two 24_M (see Appendix A). If not small, the couplings λ_{15} and $Y_{24_M}^x$ ($x = S, A$) can easily spoil the gauge coupling unification. For this reason, in this analysis we take them to be vanishing, and therefore, the mass term for the mediators 15_H and 24_M is approximately given

[*)] The superpotentials $W_{\rm ssw\,II}$ and $W_{\rm ssw\,III}$ listed in Appendix E, which differ from $W'_{\rm ssw\,I}$ and $W'_{\rm ssw\,II}$ of Eq. (2·41), are obtained from the following $W_{\rm 15H}$ and $W_{\rm 24M}$ by integrating out the triplet Higgs fields H_U^C, and by taking the limit $Y_{24_M}^{S,A} = 0$ in $W_{\rm ssw\,III}$.

by the second term in $W_{15\mathrm{H}}$ and $W_{24\mathrm{M}}$, respectively. The couplings Y_N^{I} and Y_N^{III} are generic complex matrices, Y_N^{II} is a symmetric complex one, with, respectively, eighteen and twelve irreducible parameters.

When expanding these terms in their SM representations (see Appendix A for details), in the SM basis of Eq. (2·6), these Yukawa interactions become:

$$\begin{aligned}
-\,N^c Y_N^{\mathrm{I}} \bar{5}_M 5_H &\to N^c Y_\nu^{\mathrm{I}} L H_u - e^{i\phi_l} N^c \left(Y_\nu^{\mathrm{I}} P_l\right) D^c H_U^C, \\
\bar{5}_M Y_N^{\mathrm{II}} 15_H \bar{5}_M &\to L Y_\nu^{\mathrm{II}} T L - \sqrt{2} e^{i\phi_l} D^c \left(P_l Y_\nu^{\mathrm{II}}\right) L Q_{15} \\
&\qquad + e^{i2\phi_l} D^c \left(P_l Y_\nu^{\mathrm{II}} P_l\right) S D^c, \\
5_H 24_M Y_N^{\mathrm{III}} \bar{5}_M &\to H_u W_M Y_\nu^{\mathrm{III}} L - \sqrt{\frac{6}{5}} \frac{1}{2} H_u B_M Y_\nu^{\mathrm{III}} L \\
&\qquad + e^{i\phi_l} H_u \bar{X}_M Y_\nu^{\mathrm{III}} P_l D^c + e^{i\phi_l} H_U^C G_M Y_\nu^{\mathrm{III}} P_l D^c \\
&\qquad + e^{i\phi_l} \sqrt{\frac{6}{5}} \frac{1}{3} H_U^C B_M Y_\nu^{\mathrm{III}} P_l D^c + H_U^C X_M Y_\nu^{\mathrm{III}} L. \qquad (2\cdot 47)
\end{aligned}$$

Notice that the couplings Y_N^i $(i = \mathrm{I}, \mathrm{II}, \mathrm{III})$ are substituted by the couplings Y_ν^i of Eqs. (2·40), (2·44), and (2·45), which have less independent parameters, as three phases were removed by using the three unfixed phases $e^{i\phi_l} P_l$ in Eq. (2·6). The dependence on these phases, however, is not totally eliminated. They reappear in terms containing either the triplet Higgs field, H_U^C, or a component of the seesaw mediators, and, depending on the type of seesaw implemented, are to be identified with the phases $e^{i\phi_i} P_i$ $(i = \mathrm{I}, \mathrm{II}, \mathrm{III})$ of Appendix B.

One feature of the type III seesaw mechanism becomes very evident through the expansion of the term $5_H 24_M Y_N^{\mathrm{III}} \bar{5}_M$. The components B_M interact with the components of the fields 5_H and $\bar{5}_M$ as the RHNs N^c, except for different group factors. Thus, the implementation of a seesaw of type III in the MSSU(5) model is connected to that of type I, and the relation of Eq. (2·42), in which the superscript label is now somewhat of a misnomer, must be modified:

$$m_\nu = \kappa v_u^2 = \left[\frac{1}{2} \left(Y_\nu^{\mathrm{III}}\right)^T \frac{1}{M_{W_M}} \left(Y_\nu^{\mathrm{III}}\right) + \frac{3}{10} \left(Y_\nu^{\mathrm{III}}\right)^T \frac{1}{M_{B_M}} \left(Y_\nu^{\mathrm{III}}\right)\right] v_u^2. \qquad (2\cdot 48)$$

For simplicity the same symbol Y_ν^{III} is used for the couplings in the two terms, but in reality these differ by small SU(5)-breaking corrections.

Last, but not least, these decompositions show clearly that the Yukawa couplings $Y_\nu^{\mathrm{I,II,III}}$ induce flavour violations in both, the scalar sector of leptons and right-handed down quarks, in a related way. Differently than in seesaw of type I, where the second interaction in the first decomposition decouples at M_{GUT}, in the seesaw of type II, all three interaction in the second decomposition remain active down to M_{ssw}, which differ by at least a couple of orders of magnitude from M_{GUT}. The situation is more complicated in the case of the seesaw of type III. The consequences of this feature will be discussed in Sec. 11.

We close this section by giving also the soft part of the scalar potential for the

three types of seesaw mechanism [*)]:

$$\widetilde{V}_{\rm RHN} = \Big\{ -\widetilde{N^c} A_N^{\rm I} \widetilde{\bar{5}}_M 5_H + \frac{1}{2} \widetilde{N^c} B_N \widetilde{N^c} + \text{ H.c.} \Big\} + \widetilde{N^c} \widetilde{m}^2_{N^c} \widetilde{N^c}^*,$$

$$\begin{aligned} \widetilde{V}_{\rm 15H} = \Big\{ & \frac{1}{\sqrt{2}} \widetilde{\bar{5}}_M A_N^{\rm II} 15_H \widetilde{\bar{5}}_M + B_{15} 15_H \overline{15}_H \\ & + \frac{1}{\sqrt{2}} A_{\lambda_D} \bar{5}_H 15_H \bar{5}_H + \frac{1}{\sqrt{2}} A_{\lambda_U} 5_H \overline{15}_H 5_H + A_{\lambda_{15}} 15_H 24_H \overline{15}_H + \text{ H.c.} \Big\} \\ & + \widetilde{m}^2_{15_H} 15_H^* 15_H + \widetilde{m}^2_{\overline{15}_H} \overline{15}_H^* \overline{15}_H, \end{aligned} \tag{2·49}$$

$$\begin{aligned} \widetilde{V}_{\rm 24M} = \Big\{ & 5_H \widetilde{24}_M A_N^{\rm III} \widetilde{\bar{5}}_M + \frac{1}{2} \widetilde{24}_M B_{24_M} \widetilde{24}_M \\ & + \frac{1}{2} \sum_{x=S,A} \Big(\widetilde{24}_M A^x_{24_M} \widetilde{24}_M \Big)_x 24_H \quad + \text{ H.c.} \Big\} + \widetilde{24}_M \widetilde{m}^2_{24_M} \widetilde{24}^*_M. \end{aligned}$$

§3. Effective couplings at the tree level in the SUSY-conserving sector

Nonrenormalizable operators exist in every effective model, and in general all NROs allowed by the symmetries of the model, do appear. The peculiarity of GUT models in this respect is that they postulate the existence of a large scale not far from $M_{\rm cut}$. Therefore, NROs in these models, can be far less suppressed than, say, in the SM.

Those that immediately come to mind in the context of the MSSU(5) model are the NROs obtained from renormalizable operators in which the field 24_H is inserted in all possible ways. Neglecting for a moment the dynamical part of 24_H, it is easy to see that the large *vevs* of its component B_H, in this discussion safely approximated by v_{24} and F_{24}, can partially compensate the huge suppression coming from inverse powers of $M_{\rm cut}$. The result is a milder suppression factor:

$$s \equiv \frac{v_{24}}{M_{\rm cut}}, \tag{3·1}$$

a quantity of order 10^{-2}, or larger if the GUT scale is closer to $M_{\rm cut}$. This is too large to assume that NROs can be neglected. Since it is difficult to explain theoretically why their coefficients are exactly vanishing or very small, NROs should actually be considered as an integral part of the MSSU(5) model itself.

In this section we discuss NROs of this type in the SUSY-conserving sector of our theory. We postpone the study of those in the SUSY-breaking sector to Sec. 6. Nonrenormalizable operators that do not contain the adjoint field 24_H can also play an important role for low-energy physics, for example suppressing the decay rate of the proton, and will be introduced at the end of Sec. 3.1. A discussion of the consequences that they may have for sFVs is postponed to Sec. 5.

[*)] The conventions for the soft mass terms of the fields N^c and 24_M, are discussed below Eq. (E·25) in Appendix E.

3.1. *Effective Yukawa couplings*

Nonrenormalizable operators in the Yukawa sector of the superpotential are obtained by inserting in all possible ways powers of the field 24_H for example in the two Yukawa operators of Eq. (2·2). We obtain the two classes of NROs Op^5 and Op^{10}, with Op^5 given by:

$$Op^5 = \sum_{n+m=0}^{k} \sqrt{2}\; \bar{5}_M\, C^5_{n,m} \left(\frac{24^T_H}{M_{\rm cut}}\right)^n 10_M \left(\frac{24_H}{M_{\rm cut}}\right)^m \bar{5}_H, \tag{3·2}$$

where 24^T_H is the transpose of 24_H. For the explicit expression of Op^{10}, with coefficients $C^{10}_{n_1,n_2,n_3,n_4,n_5}$, we refer the reader to Appendix A. The NROs with n, m, and n_i simultaneously vanishing in Op^5 and Op^{10} are the renormalizable ones listed in Eq. (2·2), with $C^5_{0,0} \equiv Y^5$ and $C^{10}_{0,0,0,0,0} \equiv Y^{10}$. We shall sometimes refer to them as $Op^5|_4$ and $Op^{10}|_4$.

In general, there exist other NROs that differ from these by the trace of a product of an arbitrary number of 24_H fields, duly suppressed by the same number of factors $1/M_{\rm cut}$: $Op^{i\,\prime} = Op^i \times \prod_j {\rm Tr}(24_H/M_{\rm cut})^{k_j} (i = 5, 10)$. The corrections that they induce to the various Yukawa couplings can be reabsorbed into those coming from Op^5 and Op^{10}, once the field 24_H is replaced by its *vev*, and are therefore neglected in the following discussion.

The dimension-five operators often discussed in the literature are

$$\begin{aligned} Op^5|_5 &= \frac{\sqrt{2}}{M_{\rm cut}} \left(\bar{5}_M\, C^5_{1,0}\, 24^T_H 10_M \bar{5}_H \;+\; \bar{5}_M\, C^5_{0,1}\, 10_M 24_H \bar{5}_H\right), \\ Op^{10}|_5 &= -\frac{1}{4M_{\rm cut}} \left(10_M\, C^{10}_A 10_M\, 24_H\, 5_H \;+\; 10_M\, C^{10}_S 10_M 5_H\, 24_H\right), \end{aligned} \tag{3·3}$$

where C^{10}_A and C^{10}_S corresponds to linear combinations of some coefficients of Op^{10}:

$$\begin{aligned} C^{10}_A &= \left[C^{10\,T}_{1,0,0,0,0} + C^{10\,T}_{0,1,0,0,0} + C^{10}_{0,0,1,0,0} + C^{10}_{0,0,0,1,0}\right]_A, \\ C^{10}_S &= C^{10}_{0,0,0,0,1} - \left[C^{10\,T}_{1,0,0,0,0} + C^{10\,T}_{0,1,0,0,0} + C^{10}_{0,0,1,0,0} + C^{10}_{0,0,0,1,0}\right]_S, \end{aligned} \tag{3·4}$$

and the indices S and A denote the symmetrized and antisymmetrized form of the matrices inside parentheses. The matrix C^{10}_A is then antisymmetric. It corresponds to the coupling for the irreducible interaction between the two fields 10_M combined into a $\overline{45}$ representation of SU(5) and the fields 5_H and 24_H, combined into a 45. We remind that $10 \times 10 = \bar{5}_S + \overline{45}_A + \overline{50}_S$ and $5 \times 24 = 5 + 45 + 70$. The matrix C^{10}_S is symmetric, since $C^{10}_{0,0,0,0,1}$ (like Y^{10}) is a symmetric matrix. It corresponds to the coupling for the irreducible interaction in which the two fields 10_M and the fields 5_H and 24_H combine, respectively, into a $\bar{5}$ and a 5 representations of SU(5).

As for the dimension-six NROs, $Op^5|_6$ can be easily read off from Eq. (3·2); $Op^{10}|_6$ is given in Appendix A.

Neglecting the dynamical part of the field 24_H, after the substitution $24^{24}_H \to v_{24}$, Op^5 and Op^{10} decompose into effective renormalizable operators. In particular, in

the case of Op^5, we obtain *):

$$-D^c\boldsymbol{Y}^5_D QH_d, \quad -E^c(\boldsymbol{Y}^5_E)^T LH_d, \quad -D^c\boldsymbol{Y}^5_{DU}U^c H^C_D, \quad -L\,\boldsymbol{Y}^5_{LQ}QH^C_D, \tag{3·5}$$

with couplings $\boldsymbol{Y}^5_i$ ($i = {\scriptstyle D,E,DU,LQ}$):

$$\boldsymbol{Y}^5_i \;=\; \sum_{n+m=0}^{k} \boldsymbol{Y}^5_i|_{n+m} \;=\; \sum_{n+m=0}^{k} C^5_{n,m}\, s^{(n+m)} \left(\left(I_{\bar{5}_M}\right)_i\right)^n \left(\left(I_{\bar{5}_H}\right)_i\right)^m , \tag{3·6}$$

where $(I_{\bar{5}_M})_i$ and $(I_{\bar{5}_H})_i$ collect the hypercharge values:

$$\begin{aligned}
\left(I_{\bar{5}_M}\right)_i &= \sqrt{\frac{6}{5}}\left\{+\frac{1}{3},-\frac{1}{2},+\frac{1}{3},-\frac{1}{2}\right\}, \\
\left(I_{\bar{5}_H}\right)_i &= \sqrt{\frac{6}{5}}\left\{-\frac{1}{2},-\frac{1}{2},+\frac{1}{3},+\frac{1}{3}\right\} \qquad (i = {\scriptstyle D,E,DU,LQ}).
\end{aligned} \tag{3·7}$$

In these definitions a boldface type is used to distinguish the effective Yukawa couplings from the usual ones, and all boldfaced quantities appearing hereafter are assumed to incorporate in some way the effect of NROs in which the substitution $24^{24}_H \to v_{24}$ is made. Indices are attached to these effective couplings consisting in the sequence of flavour fields in the effective renormalizable operators in which these couplings appear. We have used however $\boldsymbol{Y}^5_D$ and $\boldsymbol{Y}^{5\,T}_E$, and not $\boldsymbol{Y}^5_{DQ}$ and $\boldsymbol{Y}^{5\,T}_{LE}$, for the couplings that exist also in the MSSM, such as Y_D and Y_E. In a similar way, $\boldsymbol{Y}^{10}_U$ will be used later instead of $\boldsymbol{Y}^{10}_{UQ}$.

Specifically, up to $\mathcal{O}(s^2)$, the couplings $\boldsymbol{Y}^5_i$ are

$$\begin{aligned}
\boldsymbol{Y}^5_D &= Y^5 + \sqrt{\frac{6}{5}}\left(\frac{1}{3}C^5_{1,0} - \frac{1}{2}C^5_{0,1}\right) s + \mathcal{O}(s^2), \\
\boldsymbol{Y}^5_E &= Y^5 - \sqrt{\frac{6}{5}}\left(\frac{1}{2}C^5_{1,0} + \frac{1}{2}C^5_{0,1}\right) s + \mathcal{O}(s^2), \\
\boldsymbol{Y}^5_{DU} &= Y^5 + \sqrt{\frac{6}{5}}\left(\frac{1}{3}C^5_{1,0} + \frac{1}{3}C^5_{0,1}\right) s + \mathcal{O}(s^2), \\
\boldsymbol{Y}^5_{LQ} &= Y^5 - \sqrt{\frac{6}{5}}\left(\frac{1}{2}C^5_{1,0} - \frac{1}{3}C^5_{0,1}\right) s + \mathcal{O}(s^2),
\end{aligned} \tag{3·8}$$

from which, being $\boldsymbol{Y}^5_D - \boldsymbol{Y}^5_E = \sqrt{(5/6)}C^5_{1,0}s$, it becomes clear that only one NRO of dimension five, that with coefficient $C^5_{1,0}$, is sufficient to correct the wrong fermion spectrum of the minimal model. Once this is introduced, however, it is difficult to explain why the other NRO of dimension five, as well as NROs of higher dimensions in Op^5 should be absent. We keep our formalism as general as possible and postpone to later the issue of the level of precision in s that it is reasonable to require.

*) At the GUT threshold, the MSSM coupling Y_E has to be identified with $(\boldsymbol{Y}^5_E)^T$.

It is easy to see from this expansion that, for each element (h,k), the effective couplings $\boldsymbol{Y}^5_i$ satisfy the conditions:

$$\begin{aligned}\left|\left(\boldsymbol{Y}^5_i-\boldsymbol{Y}^5_{i'}\right)_{(h,k)}\right| &\le \mathcal{O}(s) \quad (i,i'={\scriptstyle D,E,DU,LQ}),\\ \left|\left(\boldsymbol{Y}^5_D-\boldsymbol{Y}^5_E+\boldsymbol{Y}^5_{LQ}-\boldsymbol{Y}^5_{DU}\right)_{(h,k)}\right| &\le \mathcal{O}(s^2).\end{aligned} \tag{3·9}$$

This last combination vanishes exactly if only NROs of dimension five are introduced. We refer to these relations as the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints. They help fixing, respectively, the $\mathcal{O}(1)$ and $\mathcal{O}(s)$ contributions to the effective couplings $\boldsymbol{Y}^5_i$, proving themselves a very useful tool in selecting the appropriate boundary conditions for the various $\boldsymbol{Y}^5_i$. Notice that there are no $\mathcal{O}(s^3)$-constraints. These constraints, like all the others listed in this section, as well as those in Secs. 6.1 and 7.1 for effective trilinear couplings, are derived in SU(5)-symmetric bases.

The effective renormalizable operators induced by Op^{10}, with effective couplings $\boldsymbol{Y}^{10}_j$ labelled by the index $j=\{{\scriptstyle U,UE,QQ}\}$, are

$$U^c\boldsymbol{Y}^{10}_U QH_u, \quad U^c\boldsymbol{Y}^{10}_{UE}E^cH^C_U, \quad \frac{1}{2}Q\,\boldsymbol{Y}^{10}_{QQ}QH^C_U. \tag{3·10}$$

Because of its complexity, we refrain in this case from giving general expressions of the effective couplings $\boldsymbol{Y}^{10}_j$ in terms of the coefficients appearing in Op^{10}. We give explicitly only the expansion up to $\mathcal{O}(s^2)$, the counterpart of Eq. (3·8) for $\boldsymbol{Y}^5_i$:

$$\begin{aligned}\boldsymbol{Y}^{10}_U &= Y^{10}-\sqrt{\frac{6}{5}}\left(\frac{1}{2}C^{10}_S-\frac{5}{6}C^{10}_A\right)s+\mathcal{O}(s^2),\\ \boldsymbol{Y}^{10}_{QQ} &= Y^{10}+\sqrt{\frac{6}{5}}\left(\frac{1}{3}C^{10}_S\right)s+\mathcal{O}(s^2),\\ \boldsymbol{Y}^{10}_{UE} &= Y^{10}+\sqrt{\frac{6}{5}}\left(\frac{1}{3}C^{10}_S+\frac{5}{3}C^{10}_A\right)s+\mathcal{O}(s^2).\end{aligned} \tag{3·11}$$

The effective coupling $\boldsymbol{Y}^{10}_{QQ}$ is symmetric, whereas $\boldsymbol{Y}^{10}_U$ and $\boldsymbol{Y}^{10}_{UE}$ have both a symmetric and an antisymmetric part.

The corresponding $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints for these couplings are

$$\begin{aligned}\left|\left(\boldsymbol{Y}^{10}_j-\boldsymbol{Y}^{10}_{j'}\right)_{(h,k)}\right| &\le \mathcal{O}(s) \quad (j,j'={\scriptstyle U,UE,QQ}),\\ \left|\left(\boldsymbol{Y}^{10}_{QQ}-\left(\boldsymbol{Y}^{10}_{UE}\right)^S\right)_{(h,k)}\right| &\le \mathcal{O}(s^2),\\ \left|\left(2\left(\boldsymbol{Y}^{10}_U\right)^A-\left(\boldsymbol{Y}^{10}_{UE}\right)^A\right)_{(h,k)}\right| &\le \mathcal{O}(s^2),\end{aligned} \tag{3·12}$$

where, as in the case of the effective couplings $\boldsymbol{Y}^5_i$, the left-hand side of the two $\mathcal{O}(s^2)$-constraints vanishes identically if NROs of dimension six and higher in Op^{10} are vanishing. Also in this case, the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints fix or help fixing the contributions of $\mathcal{O}(1)$ and of $\mathcal{O}(s)$ to the effective couplings $\boldsymbol{Y}^{10}_j$, for which there are no $\mathcal{O}(s^3)$-constraints.

Nonrenormalizable operators in the seesaw Yukawa sector are less relevant than those discussed until now, which can give $\mathcal{O}(1)$ corrections to the Yukawa couplings of first- and second-generation fermions. Since in this context we expect the renormalizable Yukawa couplings in the seesaw sector to be rather large, NROs with "natural" coefficients $\lesssim \mathcal{O}(1)$ turn out to play subleading roles. That remains true, of course, as far as s is $\sim 10^{-2}$ and the seesaw scale is $\sim 10^{14}\,$GeV. (The situation could, however, change if the values of s and/or of $M_{\rm ssw}$ were modified even by a not too large amount.) We consider, nevertheless, NROs also in the seesaw sectors, in order to have a description of all NROs relevant for flavour violation at the same order in s.

By inserting powers of the field 24_H in the first term of $W_{\rm RHN}$ in Eq. (2·46), we obtain the class of operators $Op^{N_{\rm I}}$ relevant for the seesaw of type I:

$$Op^{N_{\rm I}} = -\sum_{n=0}^{k} N^c\, C_n^{(N_{\rm I})}\, \bar{5}_M \left(\frac{24_H^T}{M_{\rm cut}}\right)^n 5_H, \tag{3·13}$$

with $C_0^{(N_{\rm I})} \equiv Y_N^{\rm I}$. We do not include NROs for the mass term $(1/2)N^c M_N N^c$. After the substitution $24_H^{24} \to v_{24}$, $Op^{N_{\rm I}}$ decomposes in the effective renormalizable operators:

$$N^c \boldsymbol{Y}_N^{\rm I} L H_u, \quad -N^c \boldsymbol{Y}_{ND}^{\rm I} D^c H_U^C, \tag{3·14}$$

with effective couplings defined as:

$$\boldsymbol{Y}_h^{\rm I} \;=\; \sum_{n=0}^{k} \boldsymbol{Y}_h^{\rm I}|_n \;=\; \sum_{n=0}^{k} C_n^{N_{\rm I}}\, s^n \left((-I_{5_H})_h\right)^n \qquad (h = {\scriptstyle N,ND}), \tag{3·15}$$

in which the hypercharge factors in $(I_{5_H})_h$ are

$$(-I_{5_H})_h = \sqrt{\frac{6}{5}} \left\{-\frac{1}{2}, \frac{1}{3}\right\} \qquad (h = {\scriptstyle N,ND}). \tag{3·16}$$

Thus, up to $\mathcal{O}(s^2)$, $\boldsymbol{Y}_N^{\rm I}$ and $\boldsymbol{Y}_{ND}^{\rm I}$ have the very simple expression:

$$\begin{aligned} \boldsymbol{Y}_N^{\rm I} &= Y_N^{\rm I} - \sqrt{\frac{6}{5}}\frac{1}{2} C_1^{N_{\rm I}}\, s + \mathcal{O}(s^2), \\ \boldsymbol{Y}_{ND}^{\rm I} &= Y_N^{\rm I} + \sqrt{\frac{6}{5}}\frac{1}{3} C_1^{N_{\rm I}}\, s + \mathcal{O}(s^2), \end{aligned} \tag{3·17}$$

and must satisfy only one constraint:

$$\left|\left(\boldsymbol{Y}_N^{\rm I} - \boldsymbol{Y}_{ND}^{\rm I}\right)_{(h,k)}\right| \;\leq\; \mathcal{O}(s). \tag{3·18}$$

There are no $\mathcal{O}(s^2)$- and $\mathcal{O}(s^3)$-constraints on the effective couplings $\boldsymbol{Y}_h^{\rm I}$, while $\mathcal{O}(s^2)$-constraints exist in the case of the other two seesaw types.

The treatment for the seesaw of type II and III is not explicitly reported here, as it is completely straightforward. As in type I, only the first interaction term in $W_{\rm 15H}$

and the first in $W_{24\mathrm{M}}$ are generalized to the classes of operators $Op^{N_{\mathrm{II}}}$ and $Op^{N_{\mathrm{III}}}$. We neglect NROs for the other two terms in $W_{24\mathrm{M}}$ because they involve only heavy fields. The same is true for the terms $M_{15}15_H\overline{15}_H$ and $\lambda_{15}15_H24_H\overline{15}_H$ in $W_{15\mathrm{H}}$. We actually neglect NROs for all terms of $W_{15\mathrm{H}}$, except for the first one, as they do not contain flavour fields. The small modifications that NROs would induce in the couplings of the terms involving the fields 5_H and $\bar{5}_H$ in $W_{15\mathrm{H}}$ are of no consequence for our discussion of sFVs. (See also Sec. 3.2.)

The picture of effective Yukawa couplings is particularly appealing because, for a given value k taken as upper limit of the sums in Op^5, Op^{10}, and $Op^{N_{\mathrm{I}}}$, only few linear combinations of the many coefficients, $C^5_{n,m}$, $C^{10}_{n_1,n_2,n_3,n_4,n_5}$, and $C^{N_{\mathrm{I}}}_n$, in these sums are physically relevant. These are the four, three, and two combinations defining $\boldsymbol{Y}^5_i$, $\boldsymbol{Y}^{10}_j$, and $\boldsymbol{Y}^{\mathrm{I}}_h$, respectively. For example, for the effective couplings $\boldsymbol{Y}^5_i$, the number of these combinations, four, is smaller than the number obtained summing the number of couplings of the minimal model, one, *i.e.* Y^5, plus the number of coefficients of the relevant NROs up to dimension $4+k$. This is true except in the cases $k=0,1$. For $k=0$, there is only the original coupling of the renormalizable operator, Y^5, versus the four effective couplings $\boldsymbol{Y}^5_i$, giving rise to three $\mathcal{O}(s)$-constraints. Similarly, for $k=1$ there are three original couplings, Y^5, $C^5_{1,0}$, $C^5_{0,1}$, versus four effective couplings, inducing one $\mathcal{O}(s^2)$-constraint. Clearly, there are no $\mathcal{O}(s^3)$-constraints, as the number of original couplings exceeds already the number of effective couplings. The same reasoning holds for the other two types of effective couplings $\boldsymbol{Y}^{10}_j$, and $\boldsymbol{Y}^{\mathrm{I}}_h$.

The possibility of incorporating the effects of NROs in the classes Op^5, Op^{10}, and $Op^{N_{\mathrm{I}}}$ (as well as $Op^{N_{\mathrm{II}}}$ and $Op^{N_{\mathrm{III}}}$) of dimensions large enough could be particularly important in models with $s>10^{-2}$.

The main question here is whether it is possible to promote this picture to the quantum level. If this were the case, at least to a fixed order in s, one could think of substituting the theory containing NROs with a theory including only renormalizable operators mildly but explicitly breaking SU(5), except in the gauge sector. The two theories would give the same low-energy predictions, up to tiny corrections due to terms in which, being the field 24_H dynamical, the huge $1/M_{\mathrm{cut}}$ suppression is not compensated by v_{24}. In such a case, the contributions to sFVs coming from both, seesaw operators and NROs would be automatically calculated making use of RGEs for the effective couplings.

This issue will be discussed in Sec. 10.1. We anticipate however, that in general there are some flaws in this way of reasoning. To begin with, there exist other superheavy mass parameters in $W_{\mathrm{H}}^{\mathrm{MSSU(5)}}$, such as M_5, that can compensate at the loop level the huge suppression coming from powers of $1/M_{\mathrm{cut}}$, as much as v_{24} does. Therefore, the assumption that the dynamical part of the field 24_H can be neglected when considering low-energy physics is, in general, not valid. Second, also NROs that do not depend on the field 24_H and that seem naively too suppressed to be of any relevance for low-energy physics, except for proton decay [*)], can induce corrections to the Yukawa couplings at the quantum level. Nonrenormalizable operators of this

[*)] They could have other cosmological consequences such as those discussed in Ref. 61).

type are for example those in the proton-decay class *):

$$Op^{\rm PD} = \sum_n \frac{1}{M_{\rm cut}} \left(C_n^{\rm PD}\right)^{hijk} (10_M)_h (10_M)_i (10_M)_j (\bar{5}_M)_k \left(\frac{24_H}{M_{\rm cut}}\right)^n . \qquad (3\cdot19)$$

(See details in Appendix A about the actual form of this class of operators.)

For the remainder of this section we shall continue the survey of the different NROs in the SUSY-conserving part of our model, irrespective of how they should be treated at the quantum level.

3.2. *NROs in the superpotential Higgs sector*

In addition to the NROs listed above, there are also NROs in the Higgs sector:

$$\begin{aligned} Op^{5_H} &= M_{\rm cut} \sum_{n=0}^{k+1} C_n^{5_H}\, 5_H \left(\frac{24_H}{M_{\rm cut}}\right)^n \bar{5}_H, \\ Op^{24_H} &= M_{\rm cut}^3 \sum_{\sum n_j = 2}^{k+3} C^{24_H}_{n_1,\cdots,n_j,\cdots} \prod_j \frac{1}{n_j!} {\rm Tr} \left(\frac{24_H}{M_{\rm cut}}\right)^{n_j} , \end{aligned} \qquad (3\cdot20)$$

where it is $C_0^{5_H} = M_5/M_{\rm cut}$, $C_1^{5_H} = \lambda_5$, in the first class of operators, and $C_2^{24_H} = M_{24}/M_{\rm cut}$, $C_3^{24_H} = \lambda_{24}$, in the second one. Effective couplings can be defined also in this case. For example, there are two effective couplings $(\boldsymbol{M}_5)_p$ ($p = H_U^C H_D^C, H_u H_d$) for the two operators in Eq. (A·8) in Appendix A, and six couplings $(\boldsymbol{\lambda}_5)_q$ ($q = H_U^C G_H H_D^C, \cdots, H_u B_H H_d$), for the six operators in Eq. (A·9). We do not give here their expressions in terms of the original couplings $C_n^{5_H}$. These can be easily obtained as follows:

$$\begin{aligned} (\boldsymbol{M}_5)_{H_u H d} &= \frac{\partial\, W_H^{\rm nrMSSU(5)}}{\partial H_u \partial H_d}\bigg|_{24_H^{24} \to v_{24}} = \frac{\partial\, Op^{5_H}}{\partial H_u \partial H_d}\bigg|_{24_H^{24} \to v_{24}}, \\ (\boldsymbol{\lambda}_5)_{H_u B_H H_d} &= \frac{\partial\, W_H^{\rm nrMSSU(5)}}{\partial H_u \partial B_H \partial H_d}\bigg|_{24_H^{24} \to v_{24}} = \frac{\partial\, Op^{5_H}}{\partial H_u \partial B_H \partial H_d}\bigg|_{24_H^{24} \to v_{24}}, \end{aligned} \qquad (3\cdot21)$$

where $W_H^{\rm nrMSSU(5)}$ is the Higgs sector superpotential completed with NROs.

In general, NROs in the Higgs sector induce small shifts in the vacuum condition discussed in Sec. 2.2, unless the trilinear coupling λ_{24} is small. In this case, NROs can drastically modify the vacuum structure,[44] even sizably increasing the value of v_{24}. In turn, the factor s also increases, and the effect of NROs on low-energy physics can be significantly enhanced. We keep away from such a possibility by assuming λ_{24} to be of $\mathcal{O}(1)$. Thus, the effect of Op^{5_H} and Op^{24_H} is that of producing small shifts in massive and massless parameters in the Higgs sector by ratios of $\mathcal{O}(s)$, and the vacuum structure of the minimal model can be regarded as a good approximation also when NROs in the Higgs sector are nonvanishing. We neglect these effects, which

*) If its coefficients are unsuppressed, the dimension-five operator in this class induces a too large rate for proton decay.[62]

introduce only small flavour-blind shifts in sFVs induced by Yukawa couplings, and in the same spirit we neglect also the effect of NROs in the purely-Higgs-boson sector of the seesaw of type II. (See also Sec. 10.1.)

Before closing this section, we would like to emphasize that, irrespectively of the value of λ_{24}, the introduction of NROs in the Higgs sector would not jeopardize the validity of the RGEs in Eq. (2·27) for the modified *vevs*, written in terms of the modified γ_{24} and $\tilde{\gamma}_{24}$. As shown in Appendix C, the RGEs in this equation hold quite model-independently. Thus, the approximation made here plays no role on whether the picture of effective couplings can be maintained at the quantum level.

3.3. *NROs in the SUSY-conserving part of the Kähler potential*

One of the lamented problems when dealing with NROs is that they spoil the usual requirement of minimality of the Kähler potential. Indeed, NROs such as:

$$Op^{\mathcal{K}_1}_{\bar{5}_M} = \sum_{m=1}^{k} \bar{5}_M \, C^{\mathcal{K},\bar{5}_M}_{0,m} \left(\frac{24^*_H}{M_{\rm cut}}\right)^m \bar{5}^*_M, \qquad (3\cdot 22)$$

$$Op^{\mathcal{K}_2}_{\bar{5}_M} = \sum_{\substack{n+m=2\\ n,m\neq 0}}^{k} \bar{5}_M \, C^{\mathcal{K},\bar{5}_M}_{n,m} \left(\frac{24^T_H}{M_{\rm cut}}\right)^n \left(\frac{24^*_H}{M_{\rm cut}}\right)^m \bar{5}^*_M, \qquad (3\cdot 23)$$

their Hermitian conjugates, $(Op^{\mathcal{K}_1}_{\bar{5}_M})^*$, $(Op^{\mathcal{K}_2}_{\bar{5}_M})^*$, as well as NROs as *)

$$(1/M_{\rm cut}) 5^*_H 10_M f_1 \bar{5}_M, \qquad (f_2/M^2_{\rm cut}) \bar{5}^*_M 10^*_M 10_M \bar{5}_M, \qquad (3\cdot 24)$$

do precisely that. In addition, even if accidentally vanishing at the tree level, they are induced at the quantum level by interactions in the superpotential.

In a basis with canonical kinetic terms at the renormalizable level, in which the renormalizable operator $\bar{5}_M \bar{5}^*_M$ has a coupling equal to the unit matrix, the coefficients of NROs in $Op^{\mathcal{K}_1,\bar{5}_M}$, $Op^{\mathcal{K}_2,\bar{5}_M}$, and f_1, f_2 are in general flavour dependent. Indeed, by postulating a nontrivial flavour structure of the coefficients in Op^5, Op^{10}, etc. in the superpotential, we have already assumed that the unknown dynamics generating NROs depends on flavour.

Nonrenormalizable operators of the first type, with only one antichiral or only one chiral superfield, can be reabsorbed by field redefinitions:

$$\bar{5}_M \;\to\; \bar{5}_M - \sum_{n=0}^{k} \left(C^{\mathcal{K},\bar{5}_M}_{0,m}\right)^* \left(\frac{24_H}{M_{\rm cut}}\right)^m \bar{5}_M + \cdots . \qquad (3\cdot 25)$$

Once this is done, however, it is impossible to reabsorb those of the second type through supersymmetric field redefinitions, which must preserve chirality. Thus, NROs of the second type, in general, cannot be avoided. The deviation from minimality that they produce is, however, at most of $\mathcal{O}(s^2)$.

*) For the coupling f_2 we omit both SU(5) and flavour indices.

We do not try to control in this framework the loop effects of such NROs, of $\mathcal{O}(s^2 \times s_{\rm loop})$, where $s_{\rm loop}$ is the usual loop-suppression factor:

$$s_{\rm loop} = \frac{1}{16\pi^2}, \tag{3·26}$$

accidentally of the same order of s, but, possibly, smaller in models in which $M_{\rm GUT}$ and $M_{\rm cut}$ differ by less than the usual two orders of magnitude. This is because, as will be discussed in Sec. 10, the picture of effective couplings at the quantum level can be retained if we restrict ourselves to an accuracy of $\mathcal{O}(s \times s_{\rm loop})$ for the calculation of sFVs. Thus, NROs of dimension six can contribute to sFVs, but only at the tree level, whereas those of dimension five contribute also at the one-loop level.

Among the surviving dimension-six NROs, with more than one chiral and more than one antichiral superfield, those consisting only of light fields are completely irrelevant. The operator $(f_2/M_{\rm cut}^2)\bar{5}_M^* 10_M^* 10_M \bar{5}_M$, in Eq. (3·24), for example, violates baryon number. Its contribution to the proton-decay rate, however, can be safely neglected since it is much smaller than those from the dimension-five NROs in $Op^{\rm PD}$, and from effective dimension-six operators induced by the exchange of SU(5) gauge bosons, which have a suppression factor $1/M_{\rm GUT}^2$ instead of $1/M_{\rm cut}^2$.

Other dimension-six NROs containing a pair of one chiral and one antichiral dynamical field 24_H can only affect GUT-scale physics, at least at the tree level, and are, therefore, irrelevant for our discussion. The same NROs in which the two fields 24_H are replaced by the scalar *vev* v_{24}, can certainly be removed by SU(5)-breaking field redefinitions. Whether this is true also in the case in which one or both fields 24_H acquire the *vev* F_{24} will be discussed in Sec. 8. (See in particular Eqs. (8·22)-(8·24).)

§4. Parameter counting for effective Yukawa couplings

The high-scale boundary values of the SU(5)-breaking effective Yukawa couplings must be selected in a large space of free parameters. For a correct counting of the number of those that are physical, it is convenient to express the matrices of effective Yukawa couplings in terms of their diagonalized forms and rotation matrices. Like the CKM matrix in the SM, only the matrices of mismatch between different rotation matrices are of physical relevance. Possible parametrizations for them will be discussed and the role that they play for sFVs will be highlighted.

For simplicity, we neglect NROs in the seesaw sector, as their effect, compared to the renormalizable ones with couplings of $\mathcal{O}(1)$ is expected to be much smaller than in the $\bar{5}_M$ and 10_M sectors. The generalization to the case in which they are nonvanishing is straightforward and does not add any insight to this discussion. Moreover, when considering seesaw couplings, we concentrate on the seesaw of type I. Those of type II and III can be dealt in a similar way.

We start by reviewing in more details the case of the MSSM(5) model, already outlined in Sec. 2.1 and Appendix B.

4.1. *Limit $s \to 0$*

In the case of vanishing NROs, rotations of the fields 10_M and $\bar{5}_M$ allow the reduction of Y^5 and Y^{10} to the form they have in Eq. (2·4), whereas a reduction of the independent parameters in the seesaw couplings is not possible:

$$
\begin{aligned}
Y^5 &\to \widehat{Y}^5, \\
Y^{10} &\to K_{\rm CKM}^T \widehat{Y}^{10} P_{10} K_{\rm CKM}, \\
Y_N^{\rm I} &\to e^{i\phi_{\rm I}} Y_\nu^{\rm I} P_{\rm I}.
\end{aligned} \tag{4·1}
$$

Thus, in addition to the three real and positive elements in each of the matrices $\widehat{Y}^5$ and $\widehat{Y}^{10}$, the four independent parameters of $K_{\rm CKM}$, and the fifteen elements in $Y_\nu^{\rm I}$, together with the three eigenvalues $\hat{M}_N$, all present also in the MSSM with a seesaw a type I, there are also the five physical phases in P_{10}, $P_{\rm I}$ and $\phi_{\rm I}$.

Once we decompose in their SM representations the two Yukawa operators of the minimal model, and that for the seesaw of type I, and we denote as Y_i^5 ($i = {\scriptstyle D,E,DU,LQ}$), Y_j^{10} ($j = {\scriptstyle U,UEQQ}$), and $Y_h^{\rm I}$ ($h = {\scriptstyle N,ND}$) the couplings of the resulting operators (they are the couplings of Eqs. (3·5), (3·10), and (3·15) in the limit of vanishing NROs), these additional phases can be moved to the Yukawa operators involving only colored Higgs triplets. Rotations that break SU(5) are obviously needed to achieve this. (See Eqs. (2·6) and (2·47).) We move then to a basis in which the Yukawa couplings for the $\bar{5}_M$ and 10_M sectors can be parametrized as:

$$
\begin{array}{lcllcl}
Y_D^5 & \to & \widehat{Y}^5, & Y_{DU}^5 & \to & \widehat{Y}^5 K_{\rm CKM}^\dagger P_{10}^\dagger, \\
Y_E^5 & \to & \widehat{Y}^5, & Y_{LQ}^5 & \to & e^{-i\phi_{\rm I}}\, \widehat{Y}^5 P_{\rm I}^\dagger, \\
Y_U^{10} & \to & \widehat{Y}^{10} K_{\rm CKM}, & Y_{UE}^{10} & \to & e^{i\phi_{\rm I}}\, \widehat{Y}^{10} K_{\rm CKM} P_{\rm I}, \\
 & & & Y_{QQ}^{10} & \to & K_{\rm CKM}^T \widehat{Y}^{10} P_{10} K_{\rm CKM},
\end{array} \tag{4·2}
$$

and those in the seesaw sector of type I (after diagonalization of the Majorana mass M_N) as:

$$
Y_N^{\rm I} \ \to \ Y_\nu^{\rm I}, \qquad\qquad Y_{ND}^{\rm I} \ \to \ e^{i\phi_{\rm I}} Y_\nu^{\rm I} P_{\rm I}. \tag{4·3}
$$

The determination of the boundary conditions at $M_{\rm cut}$ is, in this case, quite straightforward, if we ignore the charged lepton couplings. We fix the low-energy values of $\widehat{Y}_D$, $\widehat{Y}_U$, and $K_{\rm CKM}$ from low-energy experiment. We renormalize these couplings upwards making use of the RGEs listed in Appendix E.

At $M_{\rm ssw}$, the seesaw degrees of freedom are switched on: RHNs, in the case illustrated here. Unfortunately their coupling $Y_\nu^{\rm I}$ is not fully known. (See Eq. (2·45).) The arbitrariness by which $Y_\nu^{\rm I}$ is plagued opens up new directions in the parameter space of the problem, which can be surveyed by scanning over values of the unknown quantities, at least in principle.

At the GUT threshold, the superheavy fields are introduced. We move to a basis in which the Yukawa couplings involving only light fields, which we call now Y_E^5, Y_D^5, Y_U^{10}, and $Y_N^{\rm I}$, are as on the left columns of Eqs. (4·2) and (4·3), with Y_E^5 identified

to Y_D^5, and $Y_\nu^{\rm I}$ a fifteen parameter matrix. The couplings involving superheavy Higgs fields, Y_{DU}^5, Y_{LQ}^5, Y_{UE}^{10}, Y_{QQ}^{10} and $Y_{ND}^{\rm I}$ are then as on the right columns of the same equations, in which we have fixed the arbitrary phases P_{10}, $P_{\rm I}$, and $\phi_{\rm I}$. All fields must then be rotated from this basis to one SU(5)-symmetric, that is, one in which they can all be accommodated in the two SU(5) multiplets $\bar{5}_M$ and 10_M, with couplings for the Yukawa operators given by only three Yukawa couplings, Y^5, Y^{10}, and $Y_N^{\rm I}$. The basis in which these couplings are those of Eq. (4·1) is reached with rotations opposite to those in Eq. (2·6), in which ϕ_l and P_l are $\phi_{\rm I}$ and $P_{\rm I}$, respectively. The resulting Yukawa couplings can then be finally evolved upwards through RGEs, whose solutions at $M_{\rm cut}$ provide the high-scale boundary conditions for our problem.

4.2. *Nonvanishing NROs*

When NROs are present, the rotation matrices needed for the diagonalization of different effective Yukawa couplings induced by the same class of NROs are no longer common, and the elimination of unphysical degrees of freedom is more involved.

The effective Yukawa couplings $\boldsymbol{Y}_i^5$ and $\boldsymbol{Y}_j^{10}$ are diagonalized as:

$$\begin{aligned} \boldsymbol{Y}_i^5 &= (V_{5\,i}^\dagger)^T\, \widehat{\boldsymbol{Y}}_i^5 V_{10\,i}^\dagger \qquad (i = {\scriptstyle D,E,DU,LQ}), \\ \boldsymbol{Y}_j^{10} &= (W_{10\,j}'^\dagger)^T\, \widehat{\boldsymbol{Y}}_j^{10} W_{10\,j}^\dagger \qquad (j = {\scriptstyle U,UE,QQ}), \end{aligned} \tag{4·4}$$

with the elements of $\widehat{\boldsymbol{Y}}_D^5$, $\widehat{\boldsymbol{Y}}_E^5$, and $\widehat{\boldsymbol{Y}}_U^{10}$ giving rise to the correct fermion spectrum, *i.e.* $\mathrm{diag}(y_d, y_s, y_b)$, $\mathrm{diag}(y_e, y_\mu, y_\tau)$, and $\mathrm{diag}(y_u, y_c, y_t)$, respectively. We have left the labels 5 and 10 in the diagonalization matrices $V_{5\,i}$, $V_{10\,j}$, and $W_{10\,j}$ as a reminder of the case without NROs. The matrices $W_{10\,j}'$ were introduced because the couplings $\boldsymbol{Y}_U^{10}$ and $\boldsymbol{Y}_{UE}^{10}$ are, in general, not symmetric. For $j = {\scriptstyle QQ}$, it is $W_{10\,QQ}' = W_{10\,QQ}$.

Two of the rotation matrices in Eq. (4·4) can be absorbed by SU(5)-symmetric field redefinitions of 10_M and $\bar{5}_M$ similar to those made in the case of vanishing NROs. We choose to eliminate $V_{10\,D}^\dagger$ and $V_{5\,E}^\dagger$. We also rotate away the phases $P_{10}^{(1)}$ and $P_{10}^{(2)}$ that appear in the parametrization of the product of diagonalization matrices from which emerges now the CKM matrix: $U_{\rm CKM} = W_{10\,U}^\dagger V_{10\,D} = P_{10}^{(1)} K_{\rm CKM} P_{10}^{(2)} e^{i\phi_{10}}$. We obtain:

$$\begin{aligned} \boldsymbol{Y}_i^5 &\to (\Delta V_{5\,i}^\dagger)^T\, \widehat{\boldsymbol{Y}}_i^5\, \Delta V_{10\,i}^\dagger \qquad (i = {\scriptstyle D,E,DU,LQ}), \\ \boldsymbol{Y}_j^{10} &\to \left[(\Delta W_{10\,j}'^\dagger) K_{\rm CKM}\right]^T\, \widehat{\boldsymbol{Y}}_j^{10} P_{10} \left[\Delta W_{10\,j}^\dagger K_{\rm CKM}\right] \qquad (j = {\scriptstyle U,UE,QQ}), \end{aligned} \tag{4·5}$$

where the ten matrices of diagonalization mismatch, besides $K_{\rm CKM}$, are

$$\begin{aligned} \Delta V_{5\,i}^\dagger &= P_{10}^{(2)\dagger} V_{5\,i}^\dagger V_{5\,E} P_{10}^{(2)} \qquad (i = {\scriptstyle D,DU,LQ}), \\ \Delta V_{10\,i}^\dagger &= P_{10}^{(2)} V_{10\,i}^\dagger V_{10\,D} P_{10}^{(2)\dagger} \qquad (i = {\scriptstyle E,DU,LQ}), \\ \Delta W_{10\,j}'^\dagger &= P_{10}^{(1)\dagger} W_{10\,j}'^\dagger W_{10\,U} P_{10}^{(1)} \qquad (j = {\scriptstyle U,UE}), \\ \Delta W_{10\,j}^\dagger &= P_{10}^{(1)\dagger} W_{10\,j}^\dagger W_{10\,U} P_{10}^{(1)} \qquad (j = {\scriptstyle UE,QQ}). \end{aligned} \tag{4·6}$$

Notice that $\Delta V^{\dagger}_{5\,E}$, $\Delta V^{\dagger}_{10\,D}$, and $\Delta W^{\dagger}_{10\,U}$ are not included in this list, because they are trivially equal to the unit matrix. In addition, since $W'_{10\,QQ} = W_{10\,QQ}$, it is $\Delta W'^{\dagger}_{10\,QQ} = \Delta W^{\dagger}_{10\,QQ}$. In the limit of vanishing NROs, all these mismatch matrices reduce to the unit matrix.

All ten of them seem necessary in order to parametrize the effective Yukawa couplings $\boldsymbol{Y}^5_i$ and $\boldsymbol{Y}^{10}_j$, together with $\widehat{\boldsymbol{Y}}^5_D$, $\widehat{\boldsymbol{Y}}^5_E$, $\widehat{\boldsymbol{Y}}^{10}_U$, and $K_{\rm CKM}$, and the diagonal matrices $\widehat{\boldsymbol{Y}}^5_{DU}$, $\widehat{\boldsymbol{Y}}^5_{LQ}$, $\widehat{\boldsymbol{Y}}^{10}_{UE}$, and $\widehat{\boldsymbol{Y}}^{10}_{QQ}$, also unknown, at least at $\mathcal{O}(s)$. As shown in Appendix B, in a basis in which the Majorana mass M_N is diagonal, the Yukawa couplings for the seesaw sector are parametrized as:

$$\boldsymbol{Y}^{\rm I}_N \;=\; \boldsymbol{Y}^{\rm I}_{ND} \;\to\; e^{i\phi_{\rm I}}\, Y^{\rm I}_\nu\, P_{\rm I}, \tag{4·7}$$

where $Y^{\rm I}_\nu$ is the coupling of the minimal case.

It is easy to see, however, that in a basis reached through SU(5)-breaking rotations, in which $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^{10}_U$, and $\boldsymbol{Y}^{\rm I}_N$ match the corresponding couplings of the MSSM with a seesaw sector of type I:

$$\begin{aligned}
\boldsymbol{Y}^5_D &\to \widehat{\boldsymbol{Y}}^5_D, \\
\boldsymbol{Y}^5_E &\to \widehat{\boldsymbol{Y}}^5_E, \\
\boldsymbol{Y}^{10}_U &\to \widehat{\boldsymbol{Y}}^{10}_U K_{\rm CKM}, \\
\boldsymbol{Y}^{\rm I}_N &\to Y^{\rm I}_\nu,
\end{aligned} \tag{4·8}$$

fewer matrices are needed to parametrize the remaining couplings:

$$\begin{aligned}
\boldsymbol{Y}^5_{DU} &\to \left[\Delta V^{\dagger}_{5\,DU}\Delta V_{5\,D}\right]^T \widehat{\boldsymbol{Y}}^5_{DU} \left[\Delta V^{\dagger}_{10\,DU} K^{\dagger}_{\rm CKM} \Delta W'_{10\,U} P^{\dagger}_{10}\right], \\
\boldsymbol{Y}^5_{LQ} &\to e^{-i\phi_{\rm I}} \left[\Delta V^{\dagger}_{5\,LQ} P^{\dagger}_{\rm I}\right]^T \widehat{\boldsymbol{Y}}^5_{LQ} \left[\Delta V^{\dagger}_{10\,LQ}\right], \\
\boldsymbol{Y}^{10}_{UE} &\to e^{i\phi_{\rm I}} \left[\Delta W'^{\dagger}_{10\,UE}\Delta W'_{10\,U}\right]^T \widehat{\boldsymbol{Y}}^{10}_{UE} \left[\Delta W^{\dagger}_{10\,UE} K_{\rm CKM} \Delta V_{10\,E} P_{\rm I}\right], \\
\boldsymbol{Y}^{10}_{QQ} &\to \left[\Delta W^{\dagger}_{10\,QQ} K_{\rm CKM}\right]^T \widehat{\boldsymbol{Y}}^{10}_{QQ} P_{10} \left[\Delta W^{\dagger}_{10\,QQ} K_{\rm CKM}\right].
\end{aligned} \tag{4·9}$$

Notice that the mismatch matrix $\Delta V_{5\,D}$ was not eliminated, but only shifted to the seesaw sector:

$$\boldsymbol{Y}^{\rm I}_{ND} \;\to\; e^{i\phi_{\rm I}} Y^{\rm I}_\nu\, P_{\rm I} \Delta V_{5\,D}. \tag{4·10}$$

It turns out to be directly responsible for breaking the correlation between the seesaw-induced sQFVs and sLFVs of the minimal model. The shift described here is not peculiar of the seesaw of type I, but it is common also to the seesaw of types II and III, which have at least one operator coupling the field D^c to a seesaw-mediator field.

In this basis only eight matrices in addition to $K_{\rm CKM}$ are sufficient to parametrize all Yukawa couplings, compared to the ten needed in the basis of Eqs. (4·5) and (4·7). The remaining two matrices are shifted from the Yukawa sector to the interactions of

the fields X and $\bar{X}$ in the gauge sector, which do not affect the physics studied here. *) Since the RGEs have a covariant form under flavour rotations and their results do not depend on the basis chosen, the dependence on these two matrices present when working in the previous basis (obtained without SU(5)-breaking rotations) will have to drop out of the RGEs results.

In this analysis, we consistently use the basis in Eqs. (4·5) and (4·7). This is because, in the basis of Eqs. (4·8), (4·9), and (4·10), the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints stop having the simple form they had in bases obtained without SU(5)-breaking rotations, in which they were derived, and depend, in general, on the mismatch matrices eliminated to obtain $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^{10}_U$, and $\boldsymbol{Y}^{\rm I}_N$ in the basis of Eq. (4·8).

The determination of the boundary conditions of the effective Yukawa couplings at $M_{\rm cut}$ differs from that for the couplings of the minimal model in the following way. To begin with, $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^{10}_U$ and $K_{\rm KCM}$ are taken as low-energy inputs to be evolved up to $M_{\rm GUT}$, with the seesaw threshold dealt as in the minimal case, by introducing the coupling $Y^{\rm I}_\nu$. At $M_{\rm GUT}$, the resulting couplings are matched to $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^{10}_U$ of Eq. (4·5), by choosing three arbitrary unitary matrices for $\varDelta V^\dagger_{5\,D}$, $\varDelta V^\dagger_{10\,E}$, and $\varDelta W'^\dagger_{10\,U}$. The seesaw coupling $\boldsymbol{Y}^{\rm I}_N$ is obtained from $Y^{\rm I}_\nu$ attaching to it three arbitrary phases. Similarly, the couplings $\boldsymbol{Y}^5_{DU}$, $\boldsymbol{Y}^5_{LQ}$, $\boldsymbol{Y}^{10}_{UE}$, and $\boldsymbol{Y}^{10}_{QQ}$, parametrized by seven mismatch matrices and four diagonal matrices of couplings, are arbitrary. The only restriction in their selection is that, together with $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^5_D$ and $\boldsymbol{Y}^{10}_U$, they satisfy the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints of Eqs. (3·9) and (3·12). Since NROs were omitted in the seesaw sector, the coupling $\boldsymbol{Y}^{\rm I}_{ND}$ coincides with $\boldsymbol{Y}^{\rm I}_N$. (If included, it would differ from it at $\mathcal{O}(s)$, as shown by Eq. (3·18).) All these couplings are then evolved up to $M_{\rm cut}$.

Here, the boundary values for the soft terms are picked up and the full set of RGEs is evolved downwards. At the GUT threshold the two theories, the MSSU(5) with NROs and the MSSM, both with a seesaw sector, are matched again by performing SU(5)-breaking rotations that brings $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^{10}_U$ back to the form they have in Eq. (4·8). In particular, the field D^c is rotated as:

$$D^c \;\to\; \left[\varDelta V^\dagger_{5\,D}\right]^\dagger D^c \;=\; \left(P^\dagger_{\rm I}\varDelta_D\right) D^c. \tag{4·11}$$

The following evolution downwards is that of the MSSM with one of the three types of seesaw. At the electroweak scale further rotations must be performed to extract the low-energy physical parameters. These rotations bring the Yukawa couplings to have, for example, the form on the right-hand side of Eq. (4·8). Strictly speaking, the SU(5)-breaking rotations at $M_{\rm GUT}$, including that of Eq. (4·11), could be avoided, having only the final rotations at the electroweak scale. By doing this, the typical GUT degrees of freedom that are unphysical below $M_{\rm GUT}$ are not eliminated and they are formally included in the RGEs below this scale. Since the RGEs are covariant under unitary transformations, however, unphysical parameters do not play any role in the running, and the RGEs solutions do not depend on them. The dependence

*) They do affect the physics in which the gauge bosons X and $\bar{X}$ play a role, as for example proton decay, which can proceed through dimension-six operators induced integrating out these gauge bosons. See remark at the end of Sec. 5.

on the matrix Δ_D is then recovered at $M_{\rm weak}$, when the low-energy rotations are performed.

The procedure outlined here is conceptually clear, but the large number of unknown matrices to be specified makes the problem very difficult to handle. It is possible, however, that there exist some limiting cases, in which the number of parameters needed to specify these unitary matrices is smaller than the usual three, for angles, plus six, for phases. For example, some of the phases may be rotated away.

Before tackling this issue, we pause for a moment to compare our case with that of Ref. 19), where only one NRO is introduced [*]. In the SU(5)-symmetric basis, the effective Yukawa couplings are in this case

$$\begin{array}{rcll} \boldsymbol{\mathcal{Y}}_i^5 &=& Y^5 + C_{1,0}^5\, s\,(I_{\bar{5}_M})_i & \quad (i = {\scriptstyle D,E,DU,LQ}), \\ \boldsymbol{\mathcal{Y}}_j^{10} &=& Y_j^{10} = Y^{10} & \quad (j = {\scriptstyle U,UE,QQ}), \end{array} \tag{4·12}$$

denoted with calligraphic symbols to distinguish them from the general ones. The couplings $\boldsymbol{\mathcal{Y}}_j^{10}$ coincide with those of the minimal model and are all symmetric matrices. Thus, all matrices $W'^{\dagger}_{10\,j}$ and $W^{\dagger}_{10\,j}$ coincide with the rotation matrix $W^{\dagger}_{10}$ of the minimal case, and the mismatch matrices $\Delta W'^{\dagger}_{10\,j}$ and $\Delta W^{\dagger}_{10\,j}$ are trivial:

$$\Delta W'^{\dagger}_{10\,j} = \Delta W^{\dagger}_{10\,j} = \mathbf{1} \qquad (j = {\scriptstyle U,UE,QQ}). \tag{4·13}$$

Moreover, as the values of $(I_{\bar{5}_M})_i$ in Eq. (3·7) show, only two of the effective couplings $\boldsymbol{\mathcal{Y}}_i^5$ $(i = {\scriptstyle D,E,DU,LQ})$ are independent:

$$\boldsymbol{\mathcal{Y}}_D^5 \;=\; \boldsymbol{\mathcal{Y}}_{DU}^5, \qquad \boldsymbol{\mathcal{Y}}_E^5 \;=\; \boldsymbol{\mathcal{Y}}_{LQ}^5. \tag{4·14}$$

The first of these two equalities implies that $V^{\dagger}_{5\,D} = V^{\dagger}_{5\,DU}$ and $V^{\dagger}_{10\,D} = V^{\dagger}_{10\,DU}$, the second that $V^{\dagger}_{5\,E} = V^{\dagger}_{5\,LQ}$ and $V^{\dagger}_{10\,E} = V^{\dagger}_{10\,LQ}$. Therefore, not all mismatch matrices are independent or nontrivial. Indeed, it is:

$$\begin{array}{rclrcl} \Delta V^{\dagger}_{5\,D} &=& \Delta V^{\dagger}_{5\,DU}, & \qquad \Delta V^{\dagger}_{10\,DU} &=& \mathbf{1}, \\ \Delta V^{\dagger}_{10\,LQ} &=& \Delta V^{\dagger}_{10\,E}, & \qquad \Delta V^{\dagger}_{5\,LQ} &=& \mathbf{1}. \end{array} \tag{4·15}$$

The reduced form of these effective Yukawa couplings after SU(5)-conserving and SU(5)-breaking rotations can be easily obtained by simplifying the corresponding ones of the general case by using all these conditions. It is easy to see that, in this case, two are the mismatch matrices needed to parametrize the effective Yukawa couplings: $\Delta V^{\dagger}_{5\,D}$ and $\Delta V^{\dagger}_{10\,E}$, multiplied by some phase matrices. In addition, no arbitrariness exists for the boundary conditions of $\widehat{\boldsymbol{\mathcal{Y}}}^5_{DU}$, $\widehat{\boldsymbol{\mathcal{Y}}}^5_{LQ}$, $\widehat{\boldsymbol{\mathcal{Y}}}^{10}_{UE}$, and $\widehat{\boldsymbol{\mathcal{Y}}}^{10}_{QQ}$, which are linked to $\widehat{\boldsymbol{\mathcal{Y}}}^5_{D}$, $\widehat{\boldsymbol{\mathcal{Y}}}^5_{E}$, and $\widehat{Y}^{10}$, fixed by low-energy data.

[*] All NROs of dimension five in Op^5, Op^{10}, and $Op^{N_{\rm I}}$ are discussed in Ref. 19). Only one of them, however, the operator with coefficient $C^5_{1,0}$ in Op^5, is actually considered when counting the number of arbitrary but physical parameters introduced by NROs.

The apparent simplicity of this case, should not be deceiving, as the number of parameters that it involves is large. To show this, we parametrize $\Delta V_{5\,D}^{\dagger}$ and $\Delta V_{10\,E}^{\dagger}$ as usual:

$$\begin{aligned} \Delta V_{5\,D}^{\dagger} &= P_D^{(1)}\,K_D^{\dagger}\,P_D^{(2)} e^{i\phi_D}, \\ \Delta V_{10\,E}^{\dagger} &= P_E^{(1)}\,K_E^{\dagger}\,P_E^{(2)} e^{i\phi_E}, \end{aligned} \tag{4·16}$$

with CKM-like matrices K_E and K_D, and we express the effective Yukawa couplings $\boldsymbol{\mathcal{Y}}_i^5$ and $\boldsymbol{\mathcal{Y}}_j^{10}$ after SU(5)-breaking rotations as:

$$\begin{array}{lllllll} \boldsymbol{\mathcal{Y}}_D^5 & \to & \widehat{\boldsymbol{\mathcal{Y}}}_D^5, & \qquad & \boldsymbol{\mathcal{Y}}_{DU}^5 & \to & \widehat{\boldsymbol{\mathcal{Y}}}_D^5\,K_{\rm CKM}^{\dagger} P_{10}^{\dagger}, \\ \boldsymbol{\mathcal{Y}}_E^5 & \to & \widehat{\boldsymbol{\mathcal{Y}}}_E^5, & & \boldsymbol{\mathcal{Y}}_{LQ}^5 & \to & e^{-i(\phi_{\rm I}-\phi_E)}\,\widehat{\boldsymbol{\mathcal{Y}}}_E^5\,\Delta_E{}^{\dagger}, \\ \boldsymbol{\mathcal{Y}}_U^{10} & \to & \widehat{Y}^{10}\,K_{\rm CKM}, & & \boldsymbol{\mathcal{Y}}_{UE}^{10} & \to & e^{i(\phi_{\rm I}-\phi_E)}\,\widehat{Y}^{10}\,K_{\rm CKM}\,\Delta_E, \\ & & & & \boldsymbol{\mathcal{Y}}_{QQ}^{10} & \to & K_{\rm CKM}^T P_{10}\,\widehat{Y}^{10}\,K_{\rm CKM}, \end{array} \tag{4·17}$$

whereas the seesaw couplings get the form:

$$\boldsymbol{\mathcal{Y}}_N^{\rm I} \;\to\; Y_\nu^{\rm I}, \qquad\qquad \boldsymbol{\mathcal{Y}}_{ND}^{\rm I} \;\to\; e^{i(\phi_{\rm I}-\phi_D)}\,Y_\nu^{\rm I}\,\Delta_D. \tag{4·18}$$

The matrix Δ_D and $\Delta_E^{\dagger}$ have the form:

$$\Delta_E^{\dagger} = \left(P_{\rm I}^{\dagger} P_E^{(1)}\right) K_E^{\dagger}\,P_E^{(2)}, \qquad \Delta_D = \left(P_{\rm I} P_D^{(2)\dagger}\right) K_D P_D^{(1)\dagger}. \tag{4·19}$$

When compared with Eqs. (4·2) and (4·3), these equations show that by introducing only one NRO the number of physical parameters increases by two CKM-like matrices, K_E and K_D, three P-type phase matrices, with two phases each, and one overall phase, for a total of nine phases and six mixing angles. If we add also the three eigenvalues that distinguish now $\widehat{\boldsymbol{\mathcal{Y}}}_D^5$ from $\widehat{\boldsymbol{\mathcal{Y}}}_E^5$, the number of new physical parameters sums up to eighteen, which is the number of parameters of the complex matrix $C_{1,0}^5$. That is, all the new parameters due to the NRO introduced in this case are physical. This is, however, not true in general, when more NROs are present.

4.3. *Mismatch-matrices approximation*

An approximated parametrization of the mismatch matrices, with a consequent reduction of the number of physical parameters, can be obtained when i) $\tan\beta$ is large, and ii) s does not exceed the value of $\sim 10^{-2}$. This approximated parametrization is valid irrespective of the number of NROs introduced.

In the basis of Eqs. (4·5) and (4·7), it is easy to obtain from the $\mathcal{O}(s)$-constraint in Eq. (3·9), with $i = D$ and $i' = E$, that:[19)]

$$\left|(\Delta V_{5\,D}^{\dagger})_{(h,3)}\right| \lesssim \frac{s}{y_b}, \qquad \left|(\Delta V_{10\,E}^{\dagger})_{(k,3)}\right| \lesssim \frac{s}{y_\tau} \qquad (h,\,k \neq 3). \tag{4·20}$$

For the derivation of these inequalities we have made the approximation $(\widehat{\boldsymbol{Y}}_E^5)_{ii} = (\widehat{\boldsymbol{Y}}_D^5)_{ii}$, since the differences between these Yukawa couplings do not affect our estimate. We recall that all $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints were derived in an SU(5)-symmetric basis. Since for $s \sim 10^{-2}$ it is $s/y_b, s/y_\tau \sim 1/\tan\beta$, we conclude that, for

relatively large values of $\tan\beta$, the elements (1,3) and (2,3) of the matrices $\Delta V^{\dagger}_{5\,D}$ and $\Delta V^{\dagger}_{10\,E}$ are small, whereas the elements in the upper-left 2×2 sub-block are unconstrained.

Thus, in this limit, the matrix $\Delta V^{\dagger}_{5\,D}$ has one mixing angle only and four phases and we express it as:

$$\Delta V^{\dagger}_{5\,D} \;=\; P^{(1)}_D\big|_2\, K^T_D\big|_2\, P^{(2)}_D\, e^{i\phi_D}, \tag{4·21}$$

where $P^{(2)}_D$ is one of the usual two-phase P-matrices, $K_D\big|_2$ and $P^{(1)}_D\big|_2$ are, respectively, a 3×3 orthogonal matrix and a one-phase diagonal matrix with $\det P^{(1)}_D\big|_2 = 1$, of type:

$$K_D\big|_2 = \left(\begin{array}{cc|c} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \smash{0_2} \\ \hline \multicolumn{2}{c|}{0_2^T} & 1 \end{array}\right), \qquad P^{(1)}_D\big|_2 = \left(\begin{array}{cc|c} e^{i\phi^{(1)}_D} & 0 & \\ 0 & e^{-i\phi^{(1)}_D} & \smash{0_2} \\ \hline \multicolumn{2}{c|}{0_2^T} & 1 \end{array}\right). \tag{4·22}$$

The symbol 0_2 denotes here a two-component null vector. Clearly, we could have also chosen to parametrize $\Delta V^{\dagger}_{5\,D}$ as:

$$\Delta V^{\dagger}_{5\,D} \;=\; P^{(1)\,\prime}_D\, K^T_D\big|_2\, P^{(2)\,\prime}_D\big|_2\, e^{i\phi_D}. \tag{4·23}$$

The two parametrizations can be reduced to each other because a two-phase P-matrix can always be decomposed as: $P^{(2)}_D = P^{(8)}_D\, P^{(2)\,\prime}_D\big|_2$, with $P^{(2)\,\prime}_D\big|_2$ of the type shown in Eq. (4·22), and $P^{(8)}_D = \mathrm{diag}(e^{i\phi^{(8)}_D}, e^{i\phi^{(8)}_D}, e^{-2i\phi^{(8)}_D})$, which obviously commute with $K^T_D\big|_2$.

The matrices $\Delta^{\dagger}_D$ and $\Delta^{\dagger}_E$, then, can also be expressed in either one of these two forms. An explicit parametrization of Δ_D is

$$\Delta_D = \left(\begin{array}{c|c} \begin{pmatrix} \cos\theta_D\, e^{\,i\rho_D} & -\sin\theta_D\, e^{\,i\sigma_D} \\ \sin\theta_D\, e^{\,-i\sigma_D} & \cos\theta_D\, e^{\,-i\rho_D} \end{pmatrix} e^{-\frac{i}{2}\tau_D} & 0_2 \\ \hline 0_2^T & e^{\,i\tau_D} \end{array}\right). \tag{4·24}$$

The same type of approximated parametrization is also possible for all the mismatch matrices in Eq. (4·6). This can be seen by making other SU(5)-symmetric rotations through the mismatch matrices on which we want to extract information and applying the relevant $\mathcal{O}(s)$-constraints. For example, with the SU(5)-symmetric rotation $\bar{5}_M \to \Delta V_{5\,LQ}\,\bar{5}_M$, it is possible to remove $(\Delta V^{\dagger}_{5\,LQ})^T$ in the expression for $\boldsymbol{Y}^5_{LQ}$ in Eq. (4·5), reducing it to the same form that $\boldsymbol{Y}^5_E$ had when we derived the inequalities of Eq. (4·20). The role that $\Delta V^{\dagger}_{5\,D}$ had then is now played by $\Delta V^{\dagger}_{5\,D}\Delta V_{5\,LQ}$. Thus, the $\mathcal{O}(s)$-constraint in Eq. (3·9) with $i = D$ and $i' = LQ$, gives:

$$\left|(\Delta V^{\dagger}_{5\,D}\Delta V_{5\,LQ})_{(3,h)}\right| \lesssim \frac{s}{y_b}, \qquad \left|(\Delta V^{\dagger}_{10\,LQ})_{(k,3)}\right| \lesssim \frac{s}{y_\tau} \qquad (h,\, k \neq 3), \tag{4·25}$$

which allows us to conclude that also $\Delta V_{5\,LQ}$ and $\Delta V_{10\,LQ}$ can be parametrized in the same way. We have used here $(\widehat{\boldsymbol{Y}}^5_{LQ})_{ii} = (\widehat{\boldsymbol{Y}}^5_D)_{ii}$. As in the corresponding approximation made for Eq. (4·20), the differences between these elements are irrelevant

for this estimate. Thus, the same procedure can be iterated for the other mismatch matrices in Eq. (4·6), showing therefore that in the limit of s not larger than 10^{-2}, all the $\Delta V^{\dagger}_{5\,i}$ and $\Delta V^{\dagger}_{10\,i}$ are of the same type, if $\tan\beta$ is large. However, no clear hierarchy exists among the different elements of these matrices when $\tan\beta$ is not really large, even for values such as $\tan\beta \lesssim 10$. The same parametrization is valid also for the mismatch matrices $\Delta W^{\dagger}_{10\,j}$ and $\Delta W'^{\dagger}_{10\,j}$, but in this case, irrespective of the value of $\tan\beta$.

It should be stressed that the approximated form of the mismatch matrices is indeed an approximation. The elements (1,3) and (2,3) in these matrices are not vanishing, but small: all elements in these matrices are in general modified by small first–third and second–third generation mixing angles that are bounded from above by s/y_b, s/y_τ, or s/y_t. Moreover, they are not even small in absolute, for example when compared to the corresponding elements of the CKM matrix, but they are small with respect to the element of the MNS matrix. As will be discussed in Sec. 11, however, the fact that the elements (1,3) and (2,3) in these matrices are small in the sense discussed here does not imply that the first-third and second-third generation flavour transitions in the scalar sector are not affected by NROs.

When applied to the case in which only one NRO is introduced, this approximation reduces the number of arbitrary mixing angles to be specified at $M_{\rm GUT}$ from six to two.[19] The number of arbitrary phases in addition to the five already present in the minimal model, is five, one of which is an overall phase.

We would like to emphasize here that the case of only one NRO, that of dimension five with coefficient $C^5_{1,0}$ in $Op^5|_5$ is somewhat special. If the second of the two NROs in $Op^5|_5$ with coefficient $C^5_{0,1}$ is introduced, the number of physical mismatch matrices increases to four. This can be seen by taking $\Delta W'^{\dagger}_{10\,j} = \Delta W^{\dagger}_{10\,j} = \mathbf{1}$, for all j, in Eqs. (4·8) and (4·9), and by using the $\mathcal{O}(s^2)$-constraint of Eq. (3·9). Even when the approximation discussed here can be applied, there are still four arbitrary mixing angles to be inputted at $M_{\rm GUT}$, together with a fairly large number of phases.

We do not attempt counting them in this case, nor in the general one with ten mismatch matrices. Given how large the number of independent parameters is, these exercises are not particularly significant, at least at this stage. Indeed, it remains to be seen whether the constraints induced by the suppression of the proton-decay rate affect the form of the mismatch matrices. If these constraints do not reduce the number of arbitrary physical parameters, it is difficult to imagine the feasibility of a phenomenological study of sFVs that attempts to include all of them. Nevertheless, these matrices are not all at the same level when it comes to the modifications that they bring to the seesaw-induced sFVs. Thus, one reasonable simplification may be that of selecting specific directions of the very large parameter space opened up by the various mismatch matrices where these modifications are largest.

§5. Proton-decay constraints on mismatch matrices

As is well known, colored Higgs fields, even with a superheavy mass of $\mathcal{O}(M_{\rm GUT})$ give a too large contribution to the SU(5)-breaking dimension-five superpotential

operators:

$$W_{H^C}^{\rm PD} = \frac{1}{M_{H^C}} \Big[\big(C_{\rm LLLL}^{\rm PD}\big)^{hijk} (Q_h Q_i)(L_j Q_k) \; + \; \big(C_{\rm RRRR}^{\rm PD}\big)^{hijk} (U_h^c E_i^c)\big(D_j^c U_k^c\big) \Big] \,, \qquad (5\cdot1)$$

with the first ones, $(Q_h Q_i)(L_j Q_k)$, known as the LLLL-operators, the second ones, $(U_h^c E_i^c)(D_j^c U_k^c)$, as the RRRR-operators. The corresponding Wilson coefficients, before and after the SU(5)-breaking flavour rotations of Eq. (2·6) (or in the basis of Eq. (4·2)), are

$$\begin{aligned}
\big(C_{\rm LLLL}^{\rm PD}\big)^{hijk} &= \frac{1}{2} \big(Y^{10}\big)_{(h,i)} \big(Y^{5}\big)_{(j,k)} \; = \; \frac{1}{2} \Big[K_{\rm CKM}^T \widehat{Y}^{10} K_{\rm CKM} \Big]_{(h,i)} \Big[\widehat{Y}^{5} \Big]_{(j,k)} \,, \\
\big(C_{\rm RRRR}^{\rm PD}\big)^{hijk} &= \quad \big(Y^{10}\big)_{(h,i)} \big(Y^{5}\big)_{(j,k)} \; = \; \Big[\widehat{Y}^{10} K_{\rm CKM} \Big]_{(h,i)} \Big[\widehat{Y}^{5} K_{\rm CKM}^\dagger \Big]_{(j,k)} \,. \qquad (5\cdot2)
\end{aligned}$$

We have neglected here overall phases and two-phase matrices, as we shall do throughout this section. Thus, in spite of being proportional to small Yukawa couplings (some of the flavour indices are of first generation), these operators are only suppressed by one power of $1/M_{H^C}$, inducing in general a too-rapid proton decay.[46),63),64)] This is in contrast to the dimension-six operators mediated by GUT gauge bosons, also inducing proton decay, which are suppressed by two powers of superheavy masses.

Nonrenormalizable operators can be effective in suppressing the proton-decay rate because they can change the value of the Yukawa couplings involved in this calculation, without however altering the couplings that reproduce a correct fermion spectrum.[43),44)] Once NROs are introduced, the dimension-five operators induced by triplet Higgs bosons is formally as in Eq. (5·1), but the Wilson coefficients $C_{\rm LLLL}^{\rm PD}$ and $C_{\rm RRRR}^{\rm PD}$ are now replaced by boldface ones, $\boldsymbol{C}_{\rm LLLL}^{\rm PD}$ and $\boldsymbol{C}_{\rm RRRR}^{\rm PD}$, expressed in terms of effective Yukawa couplings [*)]:

$$\begin{aligned}
\big(\boldsymbol{C}_{\rm LLLL}^{\rm PD}\big)^{hijk} &= \frac{1}{2} \big(\boldsymbol{Y}_{QQ}^{10}\big)_{(h,i)} \big(\boldsymbol{Y}_{LQ}^{5}\big)_{(j,k)} \,, \\
\big(\boldsymbol{C}_{\rm RRRR}^{\rm PD}\big)^{hijk} &= \quad \big(\boldsymbol{Y}_{UE}^{10}\big)_{(h,i)} \big(\boldsymbol{Y}_{DU}^{5}\big)_{(j,k)} \,. \qquad (5\cdot3)
\end{aligned}$$

The additional flavour structure induced by the mismatch matrices present in the effective couplings, can be tuned to suppress them. How many NROs are needed for this suppression, *i.e.* at which order in s, the two series of operators Op^5 and Op^{10} can be truncated, is a question that requires a dedicated calculation, clearly beyond the scope of this paper. It is conceivable that the precision sufficient for the evaluation of sFVs, *i.e.* $\mathcal{O}(s)$ in the effective Yukawa couplings, may not be adequate to slow down the decay of the proton.

The tuning of the above Wilson coefficients can have an important feedback for the determination of sFVs. We can show this for specific values of $\boldsymbol{Y}_{LQ}^5$, $\boldsymbol{Y}_{DU}^5$, $\boldsymbol{Y}_{QQ}^{10}$, and $\boldsymbol{Y}_{UE}^{10}$ that were found in Ref. 45) to provide acceptable proton-decay rates. In

---

*) In spite of the language of effective couplings used, the material presented here is independent of the way NROs are treated at the quantum level.

one of two cases studied in this reference (the second one), just below the GUT threshold, in the MSSM, these are

$$
\begin{aligned}
\boldsymbol{Y}^{10}_{QQ} = \widehat{\boldsymbol{Y}}^{10}_{QQ} &= \mathrm{diag}(0,0,y_t),\\
\boldsymbol{Y}^{10}_{UE} = \widehat{\boldsymbol{Y}}^{10}_{UE} &= \mathrm{diag}(0,0,y_t),\\
\boldsymbol{Y}^{5}_{DU} = \widehat{\boldsymbol{Y}}^{5}_{DU} &= \mathrm{diag}(y_d-y_e,y_s-y_\mu,y_b),\\
\boldsymbol{Y}^{5}_{LQ} = \widehat{\boldsymbol{Y}}^{5}_{LQ} &= \mathrm{diag}(0,0,y_\tau),
\end{aligned}
\tag{5·4}
$$

in a basis in which the effective Yukawa couplings $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^{10}_U$ are [*)]

$$
\begin{aligned}
\boldsymbol{Y}^{5}_{D} = \widehat{\boldsymbol{Y}}^{5}_{D} &= \mathrm{diag}(y_d,y_s,y_b),\\
\boldsymbol{Y}^{5}_{E} = \widehat{\boldsymbol{Y}}^{5}_{E} &= \mathrm{diag}(y_e,y_\mu,y_\tau),\\
\boldsymbol{Y}^{10}_{U} = K^T_{\mathrm{CKM}}\,\widehat{\boldsymbol{Y}}^{10}_{U} K_{\mathrm{CKM}} &= K^T_{\mathrm{CKM}}\,\mathrm{diag}(y_u,y_c,y_t) K_{\mathrm{CKM}}.
\end{aligned}
\tag{5·5}
$$

(The seesaw couplings do not play any role in this discussion.)

Since they must be color antisymmetrized, the three Q's in the operators $QQQL$, two of which have the same SU(2) index, cannot belong to the same generation. Hence, the special form of $\boldsymbol{Y}^{10}_{QQ}$ and $\boldsymbol{Y}^{5}_{LQ}$ give a vanishing contribution to the coefficient $\boldsymbol{C}^{\mathrm{PD}}_{\mathrm{LLLL}}$. The contribution to $\boldsymbol{C}^{\mathrm{PD}}_{\mathrm{RRRR}}$, in contrast, is nonvanishing. According to the analysis of Ref. 45), this ansatz leads to a decay rate of the proton smaller than the existing experimental constraints for $\tan\beta \lesssim 12$, when the mass of the lightest stop squark, $m_{\tilde{t}_1}$, is ~ 400GeV, and the mass for the colored Higgs triplet is of $\mathcal{O}(M_{\mathrm{GUT}})$.

We remind, however, that the calculation in this reference is based on the inclusion of only dimension-five NROs in the classes Op^5 and Op^{10}. Thus, a further reduction of the proton-decay rate, which allows to relax the bound on $\tan\beta$, can be achieved with a simple modification of the above ansatz, without having to increase the value of $m_{\tilde{t}_1}$. By adding NROs of higher dimensions in Op^5, for example, which are practically irrelevant for the evaluation of sFVs, it is possible to tune the value of the element (1,1) of $\widehat{\boldsymbol{Y}}^{5}_{DU}$ in Eq. (5·4) to be vanishing. According to Ref. 45), the simultaneous vanishing of the elements (1,1) of $\widehat{\boldsymbol{Y}}^{5}_{DU}$ and $\widehat{\boldsymbol{Y}}^{10}_{UE}$ is a sufficient condition to obtain a contribution to the decay rate of the proton from RRRR operators also vanishing. (See the first ansatz used by the authors of this reference.) Clearly, it is sufficient to simply suppress the element (1,1) of $\widehat{\boldsymbol{Y}}^{5}_{DU}$ and therefore to suppress this contribution, without having to make it vanish altogether. This is somewhat irrelevant for the following discussion and, hereafter, whenever the effective couplings in Eq. (5·4) are mentioned, it is assumed that the element (1,1) of $\widehat{\boldsymbol{Y}}^{5}_{DU}$ is exactly vanishing.

The basis in Eqs. (5·4) and (5·5) can be interpreted as a basis obtained without SU(5)-breaking rotations, as it satisfies the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints of Eqs. (3·9) and (3·12). Thus, a comparison with the effective couplings in Eq. (4·5), allows to

[*)] We thank D. Emmanuel-Costa for confirming that this is indeed the basis used in Ref. 45).

conclude that the all mismatch matrices are trivial, or the inverse of $K_{\rm CKM}$ (which for the purposes of this discussion is nearly trivial), and all additional phases are vanishing. This choice, of course, selects one particular point in the large space of parameters opened up by the NROs. In this case, the predictions for the seesaw-induced sFVs do not differ from those of the minimal model, with vanishing NROs.

On the other hand, the light-field Yukawa couplings in Eq. (5·5) can also be interpreted as obtained through SU(5)-breaking rotations. Then, the effective Yukawa couplings associated to the triplet Higgs bosons $\boldsymbol{Y}^5_{DU}$, $\boldsymbol{Y}^5_{LQ}$, $\boldsymbol{Y}^{10}_{UE}$, $\boldsymbol{Y}^{10}_{QQ}$, have the complicated form:

$$\begin{aligned}
\boldsymbol{Y}^5_{DU} &\to \left[\Delta V^\dagger_{5\,DU}\Delta V_{5\,D}\right]^T \widehat{\boldsymbol{Y}}^5_{DU}\left[\Delta V^\dagger_{10\,DU}K^\dagger_{\rm CKM}\Delta W'_{10\,U}K_{\rm CKM}\right],\\
\boldsymbol{Y}^5_{LQ} &\to \left[\Delta V^\dagger_{5\,LQ}\right]^T \widehat{\boldsymbol{Y}}^5_{LQ}\left[\Delta V^\dagger_{10\,LQ}\right],\\
\boldsymbol{Y}^{10}_{UE} &\to \left[\Delta W'^\dagger_{10\,UE}\Delta W_{10\,U}K_{\rm CKM}\right]^T \widehat{\boldsymbol{Y}}^{10}_{UE}\left[\Delta W^\dagger_{10\,UE}K_{\rm CKM}\Delta V_{10\,E}\right],\\
\boldsymbol{Y}^{10}_{QQ} &\to \left[\Delta W^\dagger_{10\,QQ}K_{\rm CKM}\right]^T \widehat{\boldsymbol{Y}}^{10}_{QQ}\left[\Delta W^\dagger_{10\,QQ}K_{\rm CKM}\right],
\end{aligned}\tag{5·6}$$

where we have neglected phases, as this discussion has only demonstrative purposes. The identification of these coupling with those of Eq. (5·4) requires now:

$$\begin{aligned}
\Delta V_{5\,DU} &= \Delta V_{5\,D},\\
\Delta V_{10\,DU} &= K^\dagger_{\rm CKM}\Delta W'_{10\,U}K_{\rm CKM},\\
\Delta V_{5\,LQ} &= \Delta V_{10\,LQ} \;=\; \mathbf{1},\\
\Delta W'_{10\,UE} &= \Delta W_{10\,U}K_{\rm CKM},\\
\Delta W_{10\,UE} &= K_{\rm CKM}\Delta V_{10\,E},\\
\Delta W_{10\,QQ} &= K_{\rm CKM},
\end{aligned}\tag{5·7}$$

which fixes three of the mismatch matrices. It leaves the others still undetermined, although with some relations among them, due to the existence of the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints for effective couplings. Since the simple form for these constraints given in Eqs. (3·9) and (3·12) applies to couplings in bases obtained without SU(5)-breaking rotations, we need to obtain the original set of effective couplings in the SU(5)-conserving basis from which those in Eqs. (5·5) and (5·6) emerge. To this end, it is sufficient to insert in Eq. (4·5) the diagonal form of the effective Yukawa couplings in Eqs. (5·4) and (5·5) and the mismatch matrices in Eq. (5·7). This results in the following effective Yukawa couplings:

$$\begin{aligned}
\boldsymbol{Y}^5_{D} &\to \Delta V^{\dagger\,T}_{5\,D}\,\widehat{\boldsymbol{Y}}^5_{DU},\\
\boldsymbol{Y}^5_{E} &\to \widehat{\boldsymbol{Y}}^5_{DU}\,\Delta V^\dagger_{10\,E},\\
\boldsymbol{Y}^5_{LQ} &\to \widehat{\boldsymbol{Y}}^5_{LQ},\\
\boldsymbol{Y}^5_{DU} &\to \Delta V^{\dagger\,T}_{5\,D}\,\widehat{\boldsymbol{Y}}^5_{DU}\,\Delta V^\dagger_{10\,DU},
\end{aligned}\tag{5·8}$$

for the $\bar{5}_M$ sector, and in:

$$\begin{aligned}
&\boldsymbol{Y}_U^{10} \to \Delta V_{10\,DU}^{\dagger\,T} K_{\rm CKM}^T \,\widehat{\boldsymbol{Y}}_U^{10}\, K_{\rm CKM},\\
&\boldsymbol{Y}_{QQ}^{10} \to \widehat{\boldsymbol{Y}}_{QQ}^{10},\\
&\boldsymbol{Y}_{UE}^{10} \to \Delta V_{10\,DU}^{\dagger\,T} \,\widehat{\boldsymbol{Y}}_U^{10}\, \Delta V_{10\,E}^{\dagger},
\end{aligned} \tag{5·9}$$

for the 10_M sector.

A considerable simplification can be made at this point by applying the approximation of Sec. 4.3 to the mismatch matrices. Having established that the limitation on $\tan\beta$ obtained when using the ansatz of Eqs. (5·5) and (5·4) can be evaded with some modifications, we can make use of this approximation, which is valid only for large $\tan\beta$, at least for some of the mismatch matrices. It becomes then a simple (although tedious) exercise to work out the expressions of the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints. By neglecting the first generation Yukawa coupling y_u and the CKM mixing angles, we obtain:

$$\begin{aligned}
\left|\frac{1}{2}\,(\Delta V_{10\,DU})_{(1,2)}\, y_c\right| &\le \mathcal{O}(s^2),\\
\left|(\Delta V_{5\,D})_{(1,2)}\left\{\left[1-(\Delta V_{10\,DU})_{(2,2)}\right] y_s + (\Delta V_{10\,DU})_{(2,2)}\, y_\mu\right\}\right| &\le \mathcal{O}(s^2).
\end{aligned} \tag{5·10}$$

For the typical values that the Yukawa couplings in these expressions have at $M_{\rm GUT}$, *i.e.* $y_c \sim 10^{-3}$ and $y_s \sim y_\mu/3 \sim 10^{-4}\tan\beta$, the absolute value of $(\Delta V_{5\,D})_{(1,2)}$ cannot exceed 0.1 for $\tan\beta > 10$. Thus, the mismatch matrix $\Delta V_{5\,D}$, which coincides with Δ_D of Eqs. (4·11) and (4·10) in the limit of vanishing phases, is approximated by the unit matrix, at least as much as $K_{\rm CKM}$ is. As a consequence, the predictions for sFVs induced by the seesaw couplings do not deviate substantially from those of the MSSU(5) model with vanishing NROs.

The situation may be different if $\tan\beta$ is small. In such a case, none of the three mixing angles of the mismatch matrix $\Delta V_{5\,D}$ needs to be small and the seesaw-induced sFVs may have patterns different from those observed in the MSSU(5) model without NROs.

Moreover, we would like to emphasize that, it is not possible to claim at this point that the results obtained with the ansatz of Eq. (5·4) are generic. This specific ansatz is claimed in Ref. 45) to be only a sufficient one to suppress the proton-decay rate, but not necessary.

We have also observed that the inclusion of NROs with dimension higher than five tends to facilitate the suppression of the proton-decay rate. In contrast, the use of only one NRO made in Ref. 19), *i.e.* that with coefficient $C_{1,0}^5$ in Op^5, while adequate to obtain the correct fermion spectrum, may not be sufficient for this suppression. The two coefficients corresponding to the LLLL- and RRRR-operators are now

$$\begin{aligned}
\left(\boldsymbol{\mathcal{C}}_{\rm LLLL}^{\rm PD}\right)^{hijk} &= \frac{1}{2}\left[K_{\rm CKM}^T \widehat{Y}^{10} K_{\rm CKM}\right]_{(h,i)} \left[\widehat{\boldsymbol{\mathcal{Y}}}_E^5\, \Delta V_{10\,E}\right]_{(j,k)},\\
\left(\boldsymbol{\mathcal{C}}_{\rm RRRR}^{\rm PD}\right)^{hijk} &= \quad \left[\widehat{Y}^{10} K_{\rm CKM}\, \Delta V_{10\,E}^{\dagger}\right]_{(h,i)} \left[\widehat{\boldsymbol{\mathcal{Y}}}_D^5 K_{\rm CKM}^{\dagger}\right]_{(j,k)}.
\end{aligned} \tag{5·11}$$

Although a check should actually be made, the fact that only one mismatch matrix is present, $\Delta V_{10\,E}$, may render impossible to neutralize the effects of the dangerous products $\widehat{Y}^{10} K_{\rm CKM}$ and $K^T_{\rm CKM} \widehat{Y}^{10} K_{\rm CKM}$.

One way to suppress the decay rate of the proton that does not affect the analysis of sFVs (and that could solve the possible impasse in Ref. 19)) may be that of introducing NROs in the already mentioned class $Op^{\rm PD}$.[48] Naively thinking, the cancellation between the contributions from such operators and from the effective operators in $\boldsymbol{W}^{\rm PD}_{HC}$ seems possible. The operators in $Op^{\rm PD}$ have a stronger suppression than those in $\boldsymbol{W}^{\rm PD}_{H^c}$, as $1/M_{HC} \sim 1/M_{\rm GUT}$ is larger than $1/M_{\rm cut}$, but their coefficients can be of $\mathcal{O}(1)$, whereas $C^{\rm PD}_{\rm LLLL}$ and $C^{\rm PD}_{\rm RRRR}$ (as well as $\boldsymbol{C}^{\rm PD}_{\rm LLLL}$ and $\boldsymbol{C}^{\rm PD}_{\rm RRRR}$), are suppressed by small Yukawa couplings and CKM mixings elements. An explicit check of such a possibility should be made.

We would like to close this section with the following observation. If they are not reduced to be trivial by the tuning required to suppress the Higgs-triplet induced decay rate of the proton, some of the mismatch matrices may appear in the gauge-boson induced operators of dimension six that also induce proton decay. Their presence may actually alter the relative size of the different decay modes of the proton. It remains therefore to be checked whether $p \to e^+\pi^0$, which is the dominant mode in the MSSU(5) model, is still the dominant one also in this case.

§6. Effective couplings at the tree level in the SUSY-breaking sector

An important role is played here by the auxiliary *vev* F_{24}, overlooked in the literature devoted to the evaluation of sFVs within nrMSSU(5) models. The fact that $F_{24} \ll F_{\sf X}$, the largest SUSY-breaking auxiliary *vev* of $\mathcal{O}(\widetilde{m} M_{\rm cut})$, may perhaps induce to think that the effect of the much smaller SUSY-breaking *vev* F_{24}, of $\mathcal{O}(\widetilde{m} M_{\rm GUT})$, is negligible. In reality, the hierarchy between $F_{\sf X}$ and F_{24}:

$$\frac{F_{24}}{F_{\sf X}} = \mathcal{O}(s), \tag{6·1}$$

is of the same order of the expansion parameter in the series defining effective couplings. Thus, F_{24} turns out to be essential for the identification of the correct expression of the soft effective couplings in terms of the original parameters of the model (discussed in this section), in the determination of the boundary conditions for the soft effective couplings (to be discussed in Secs. 7 and 8), and in guaranteeing that the soft effective couplings evolve like the soft couplings of an MSSU(5) model with the SU(5) symmetry broken at $\mathcal{O}(s)$ everywhere except in the gauge sector (to be shown in Sec. 10 and Appendix D).

6.1. *Effective trilinear couplings*

One may naively think that effective trilinear couplings can be simply read off from soft operators such as:

$$\widetilde{O}p^5 = \sum_{n+m=0}^{k} \sqrt{2}\, \widetilde{\bar{5}}_M \, \widetilde{C}^5_{n,m} \left(\frac{24_H^T}{M_{\rm cut}}\right)^n \widetilde{10}_M \left(\frac{24_H}{M_{\rm cut}}\right)^m \bar{5}_H, \tag{6·2}$$

when the field 24_H^{24} is replaced by its *vev* v_{24}. These operators are obtained from the superpotential ones:

$$Op^5(\mathsf{X}) = \sum_{n+m=0}^{k} \left(\frac{\mathsf{X}}{M_{\rm cut}}\right) \sqrt{2}\, \bar{5}_M\, a^5_{n,m} C^5_{n,m} \left(\frac{24_H^T}{M_{\rm cut}}\right)^n 10_M \left(\frac{24_H}{M_{\rm cut}}\right)^m \bar{5}_H, \qquad (6\cdot3)$$

where X is the field whose auxiliary component, F_{X}, breaks SUSY. Thus, the coefficients $\widetilde{C}^5_{n,m}$ are given by:

$$\widetilde{C}^5_{n,m} = f_{\mathsf{X}}\, a^5_{n,m}\, C^5_{n,m} \equiv A^5_{n,m}\, C^5_{n,m}, \qquad (6\cdot4)$$

where f_{X} is

$$f_{\mathsf{X}} = \frac{F_{\mathsf{X}}}{M_{\rm cut}}. \qquad (6\cdot5)$$

In addition, since the *vev* F_{24} is in general nonvanishing, trilinear couplings get contributions also simply from Op^5 itself. As a consequence, the coefficient $\widetilde{C}^5_{n,m}$ gets shifted by the quantity $(n+m)\, f_{24} C^5_{n,m}$, where f_{24} is also of $\mathcal{O}(\widetilde{m})$ (see Eq. (2·28)).

As in the case of effective Yukawa couplings, effective renormalizable trilinear terms are generated when the field 24_H is replaced by its *vevs*:

$$-\,\widetilde{D}^c \boldsymbol{A}^5_D\, \widetilde{Q} H_d, \quad -\widetilde{E}^c (\boldsymbol{A}^5_E\,)^T \widetilde{L} H_d, \quad -\widetilde{D}^c \boldsymbol{A}^5_{DU} \widetilde{U}^c H^C_D, \quad -\widetilde{L}\, \boldsymbol{A}^5_{LQ} \widetilde{Q} H^C_D, \qquad (6\cdot6)$$

with couplings $\boldsymbol{A}^5_i$ ($i = {\scriptstyle D,E,DU,LQ}$) given by:

$$\begin{aligned} \boldsymbol{A}^5_i \;&=\; \boldsymbol{A}^5_{i,F_{\mathsf{X}}} + \boldsymbol{A}^5_{i,F_{24}} = \sum_{n+m=0}^{k} \boldsymbol{A}^5_i|_{n+m} = \sum_{n+m=0}^{k} \boldsymbol{A}^5_{i,F_{\mathsf{X}}}|_{n+m} + \boldsymbol{A}^5_{i,F_{24}}|_{n+m} \\ &= \sum_{n+m=0}^{k} \left[\widetilde{C}^5_{n,m} + (n+m)\, f_{24} C^5_{n,m}\right] s^{(n+m)} \left((I_{\bar{5}_M})_i\right)^n \left((I_{\bar{5}_H})_i\right)^m, \qquad (6\cdot7) \end{aligned}$$

where $\boldsymbol{A}^5_{i,F_{\mathsf{X}}}$ and $\boldsymbol{A}^5_{i,F_{24}}$ are the part generated by the auxiliary component of X and by the auxiliary component of the field 24_H, respectively. The hypercharge factors $(I_{\bar{5}_M})_i$ and $(I_{\bar{5}_H})_i$ are those of Eq. (3·7), and $\widetilde{C}^5_{0,0} = A^5_{0,0} Y^5$ is the coupling A^5 in Eq. (2·17).

Similar considerations apply also to the 10_M and the seesaw sectors, with effective trilinear parameters $\boldsymbol{A}^{10}_U$, $\boldsymbol{A}^{10}_{QQ}$, $\boldsymbol{A}^{10}_{UE}$, and $\boldsymbol{A}^{\rm I}_N$, $\boldsymbol{A}^{\rm I}_{ND}$, to be defined in analogy to the various $\boldsymbol{A}^5_i$. In an analogous way, we can also define the various effective trilinear of the Higgs sector and all effective bilinear couplings.

There are constraints on the couplings $\boldsymbol{A}^5_i$, $\boldsymbol{A}^{10}_j$ and $\boldsymbol{A}^{\rm I}_h$, completely similar to the $\mathcal{O}(s)$- and $\mathcal{O}(s^2)$-constraints on the effective Yukawa couplings. For example, the couplings $\boldsymbol{A}^5_i$ must satisfy:

$$\begin{aligned} \left|\left(\boldsymbol{A}^5_i - \boldsymbol{A}^5_{i'}\right)_{(h,k)}\right| \quad &\le \quad \mathcal{O}(s\widetilde{m}) \quad (i,i' = {\scriptstyle D,E,DU,LQ}), \\ \left|\left(\boldsymbol{A}^5_D - \boldsymbol{A}^5_E + \boldsymbol{A}^5_{LQ} - \boldsymbol{A}^5_{DU}\right)_{(h,k)}\right| \quad &\le \quad \mathcal{O}(s^2\widetilde{m}). \end{aligned} \qquad (6\cdot8)$$

In Sec. 7.1 we shall elaborate more on the form that the constraints on effective trilinear couplings can have at $M_{\rm cut}$, if we impose a certain type of universality for the soft massive parameters.

6.2. *Effective sfermion masses*

The soft mass for the field $\bar{5}_M$ arises in the MSSU(5) model from the usual Kähler potential operator:

$$Op^{\mathcal{K}_0}_{\bar{5}_M}(\mathsf{X}^*\!,\mathsf{X}) \;=\; \left(\frac{\mathsf{X}^*}{M_{\rm cut}}\right)\bar{5}_M\,|d^0_{0,0}|^2\,\bar{5}^*_M\left(\frac{\mathsf{X}}{M_{\rm cut}}\right), \tag{6·9}$$

$\widetilde{m}^2_{\bar{5}_M} = |f_{\mathsf{X}}|^2|d^0_{0,0}|^2$. To this, contributions from the class of operators:

$$Op^{\mathcal{K}_1}_{\bar{5}_M}(\mathsf{X}^*\!,\mathsf{X}) = \left(\frac{\mathsf{X}^*}{M_{\rm cut}}\right)\Big[\sum_{m=1}^{k}\bar{5}_M\,\left(d^0_{0,m}\right)^2 C^{\mathcal{K},\bar{5}_M}_{0,m}\left(\frac{24^*_H}{M_{\rm cut}}\right)^m\bar{5}^*_M\Big]\left(\frac{\mathsf{X}}{M_{\rm cut}}\right), \tag{6·10}$$

$$Op^{\mathcal{K}_2}_{\bar{5}_M}(\mathsf{X}^*\!,\mathsf{X}) = \left(\frac{\mathsf{X}^*}{M_{\rm cut}}\right)\Big[\sum_{\substack{n+m=2\\ n,m\neq 0}}^{k}\bar{5}_M\,\left(d^0_{n,m}\right)^2 C^{\mathcal{K},\bar{5}_M}_{n,m}\left(\frac{24^T_H}{M_{\rm cut}}\right)^n\left(\frac{24^*_H}{M_{\rm cut}}\right)^m\bar{5}^*_M\Big]\left(\frac{\mathsf{X}}{M_{\rm cut}}\right), \tag{6·11}$$

and their Hermitian conjugates, $(Op^{\mathcal{K}_1}_{\bar{5}_M}(\mathsf{X}^*\!,\mathsf{X}))^*$ and $(Op^{\mathcal{K}_2}_{\bar{5}_M}(\mathsf{X}^*\!,\mathsf{X}))^*$, must be added. The couplings $C^{\mathcal{K},\bar{5}_M}_{n,m}$ are matrices in flavour space as the dynamics that generates these NROs, as well as the NROs in the Yukawa sector, is not flavour blind.

Since $F_{24}\neq 0$, contributions come also from:

$$Op^{\mathcal{K}_1}_{\bar{5}_M}(\mathsf{X}) \;=\; \Big[\sum_{m=1}^{k}\bar{5}_M\,\left(d^1_{0,m}\right)^2 C^{\mathcal{K},\bar{5}_M}_{0,m}\left(\frac{24^*_H}{M_{\rm cut}}\right)^m\bar{5}^*_M\Big]\left(\frac{\mathsf{X}}{M_{\rm cut}}\right), \tag{6·12}$$

$$Op^{\mathcal{K}_2}_{\bar{5}_M}(\mathsf{X}) \;=\; \Big[\sum_{\substack{n+m=2\\ n,m\neq 0}}^{k}\bar{5}_M\,\left(d^1_{n,m}\right)^2 C^{\mathcal{K},\bar{5}_M}_{n,m}\left(\frac{24^T_H}{M_{\rm cut}}\right)^n\left(\frac{24^*_H}{M_{\rm cut}}\right)^m\bar{5}^*_M\Big]\left(\frac{\mathsf{X}}{M_{\rm cut}}\right), \tag{6·13}$$

and from $(Op^{\mathcal{K}_1}_{\bar{5}_M}(\mathsf{X}))^*$ and $(Op^{\mathcal{K}_2}_{\bar{5}_M}(\mathsf{X}))^*$, when one field 24_H gets the F_{24} *vev*, all others, the *vev* v_{24}. The upper index in the coefficients $d^1_{0,m}$ and $d^1_{n,m}$ is a reminder of this. Contributions are also due to the very same operator $Op^{\mathcal{K}_2}_{\bar{5}_M}$ of Eq. (3·23). They are obtained when one 24^T_H and one 24^*_H fields are replaced by the *vev* F_{24}, whereas all others acquire the *vev* v_{24}.

Once all the fields 24_H are replaced by their *vevs*, these operators induce a splitting between the soft mass for $\widetilde{D}^c$ and $\widetilde{L}$:

$$\widetilde{\bar{5}}_M\,\widetilde{m}^2_{\bar{5}_M}\widetilde{\bar{5}}^*_M \;\to\; \widetilde{D}^c\,\widetilde{\boldsymbol{m}}^2_{D^c}\widetilde{D}^{c\,*} + \widetilde{L}^*\,(\widetilde{\boldsymbol{m}}^2_L)^*\widetilde{L}, \tag{6·14}$$

where the two effective mass parameters $\widetilde{\boldsymbol{m}}^2_f$ ($f = D^c,L$) are

$$\widetilde{\boldsymbol{m}}^2_f \;=\; \widetilde{m}^2_{\bar{5}_M} + \Big[\widetilde{D}^0_1+\widetilde{D}^1_1\Big]s\left(I_{\bar{5}_M}\right)_f + \Big[\widetilde{D}^0_2+\widetilde{D}^1_2+\widetilde{D}^2_2\Big]s^2\big(\left(I_{\bar{5}_M}\right)_f\big)^2 + \mathcal{O}(s^3), \tag{6·15}$$

with

$$\left(I_{\bar{5}_M}\right)_f = \sqrt{\frac{6}{5}}\left\{\frac{1}{3},-\frac{1}{2}\right\} \qquad (f = D,L). \tag{6·16}$$

The Hermitian coefficients $\widetilde{D}_i^0$, $\widetilde{D}_i^1$, and $\widetilde{D}_i^2$, which get contributions from 0, 1, 2 *vevs* F_{24}, have obvious definitions in terms of the original ones. Those at $\mathcal{O}(s)$ are

$$\begin{aligned}\widetilde{D}_1^0 &= |f_{\mathsf{X}}|^2\Big[\big(d_{0,1}^0\big)^2 C_{0,1}^{\mathcal{K},\bar{5}_M} + \text{H.c.}\Big],\\ \widetilde{D}_1^1 &= \Big\{f_{\mathsf{X}} f_{24}^*\Big[\big(d_{0,1}^1\big)^2 C_{0,1}^{\mathcal{K},\bar{5}_M} + \text{H.c.}\Big] + \text{H.c.}\Big\},\end{aligned} \tag{6·17}$$

whereas those at $\mathcal{O}(s^2)$ are given by:

$$\begin{aligned}\widetilde{D}_2^0 &= |f_{\mathsf{X}}|^2\Big[\Big(\big(d_{0,2}^0\big)^2 C_{0,2}^{\mathcal{K},\bar{5}_M} + \big(d_{1,1}^0\big)^2 C_{1,1}^{\mathcal{K},\bar{5}_M}\Big) + \text{H.c.}\Big],\\ \widetilde{D}_2^1 &= \Big\{f_{\mathsf{X}} f_{24}^*\Big[\Big(2\,\big(d_{0,2}^1\big)^2 C_{0,2}^{\mathcal{K},\bar{5}_M} + \big(d_{1,1}^1\big)^2 C_{1,1}^{\mathcal{K},\bar{5}_M}\Big) + \text{H.c.}\Big] + \text{H.c.}\Big\},\\ \widetilde{D}_2^2 &= |f_{24}|^2\Big[C_{1,1}^{\mathcal{K},\bar{5}_M} + \text{H.c.}\Big].\end{aligned} \tag{6·18}$$

Nevertheless, since we want to reduce the Kähler potential to a form as close as possible to the canonical one, it is important to establish first whether NROs such as those in Eqs. (6·10), (6·11), (6·12), and (6·13) and their Hermitian conjugates can be removed as a result of field redefinitions at least when the field 24_H is nondynamical. This problem will be discussed in detail in Sec. 8.

We have concentrated here on the effective masses in the $\bar{5}_M$ sector, but analogous considerations hold also in the case of the effective masses in the 10_M sector. In the case of the seesaw sector of type I, however, the effective soft mass for RHNs coincide with the coefficient of the renormalizable mass term, $\widetilde{m}_{N^c}^2$ up to an overall factor $\prod_j \mathrm{Tr}(24_H/M_{\rm cut})^{k_j}$. This is because N^c is an SU(5) singlet. Since we neglect NROs in the Higgs sector, also the soft mass for the fields 15_H and $\bar{15}_H$, in the seesaw of type II, is the coefficient of the renormalizable mass term. In the case of the field 24_M in the seesaw of type III, in contrast, the two masses are different.

§7. Universal boundary conditions with sFVs

In the MSSU(5) model, as always when trying to highlight sFVs induced at the quantum level, the ansatz of universality and proportionality of the boundary conditions for the soft SUSY-breaking parameters is adopted. Thus, by taking, again, the seesaw of type I as case study, all soft masses squared in Eqs. (2·17) and (2·49) are assumed at $M_{\rm cut}$ to be flavour independent and all equal to the coupling $\widetilde{m}_0^2$. In the same equations, the bilinear couplings B_5, B_{24}, B_N, and the trilinear couplings A^5, A^{10}, $A_N^{\rm I}$ are such that:

$$\frac{B_5(M_{\rm cut})}{M_5(M_{\rm cut})} = \frac{B_{24}(M_{\rm cut})}{M_{24}(M_{\rm cut})} = \frac{B_N(M_{\rm cut})}{M_N(M_{\rm cut})} = B_0, \tag{7·1}$$

and

$$\frac{A^5(M_{\rm cut})}{Y^5(M_{\rm cut})} = \frac{A^{10}(M_{\rm cut})}{Y^{10}(M_{\rm cut})} = \frac{A_N^{\rm I}(M_{\rm cut})}{Y_N^{\rm I}(M_{\rm cut})} = A_0, \tag{7·2}$$

with flavour- and field-independent massive couplings B_0 and A_0. Once the boundary conditions for the superpotential massive parameters M_5, M_{24}, and M_N, are

determined at the cutoff scale, and those for the Yukawa couplings Y^5, Y^{10}, and $Y_N^{\rm I}$, are fixed in the way outlined in the previous section, those for the bilinear couplings B_5, B_{24}, and B_N, and for the trilinear ones, A^5, A^{10}, and $A_N^{\rm I}$ are also fixed.

The situation is more complicated when NROs are present.

7.1. *Effective trilinear couplings*

We consider here specifically the couplings $\boldsymbol{A}_i^5$, but the discussion holds for all effective trilinear couplings. Notice that, for a target precision of $\mathcal{O}(s \times s_{\rm loop})$ in the evaluation of sQFVs and sLFVs, the relevant terms in the expansions of $\boldsymbol{A}_i^5$ are up to $k = 2$. That is, the tree-level contribution of dimension-six NROs is needed.

A comparison of $\boldsymbol{A}_i^5$ in Eq. (6·7) with $\boldsymbol{Y}_i^5$ in Eq. (3·6) shows that the effective trilinear couplings, in general, are not aligned with the effective Yukawa couplings at $M_{\rm cut}$. Clearly, a simple extension of the conventional notion of universality used in absence of NROs, which assigns the same proportionality constant to different n-linear couplings ($n \le 3$) in operators of same dimensionality:

$$A_{1,0}^5 = A_{0,1}^5 \equiv A_1, \qquad A_{2,0}^5 = A_{0,2}^5 = A_{1,1}^5 = A_2, \qquad \cdots , \tag{7·3}$$

is not sufficient to guarantee the proportionality of $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{Y}_i^5(M_{\rm cut})$, as it leads to:

$$\boldsymbol{A}_i^5(M_{\rm cut}) = \sum_{n+m=0}^{k} \left[A_{n+m}^5 + (n+m)\, f_{24}\right] C_{n,m}^5(M_{\rm cut})\, s^{(n+m)} \left((I_{\bar{5}_M})_i\right)^n \left((I_{\bar{5}_H})_i\right)^m . \tag{7·4}$$

A more restrictive concept of universality, in which all the proportionality constants, A_0, A_1, A_2, $\cdots$, are equal can solve the problem if the auxiliary *vev* $F_{24} = 0$ vanishes at $M_{\rm cut}$, which implies:

$$f_{24} = B_0 - A_0 = 0. \tag{7·5}$$

(See Eq. (2·28).) When this condition is realized, the equality of the proportionality constant of all trilinear and n-linear couplings:

$$B_0 = A_0 = A_1 = A_2 = \cdots , \tag{7·6}$$

leads to:

$$\boldsymbol{A}_i^5(M_{\rm cut}) = A_0 \boldsymbol{Y}_i^5(M_{\rm cut}). \tag{7·7}$$

If such a notion of universality is adopted, the tuning parameter of the scalar sector, $\varDelta$, discussed in Sec. 2.2, vanishes identically. It should also be noticed that, in spite of a vanishing boundary condition at $M_{\rm cut}$, f_{24} is generated at different scales by radiative corrections, since it evolves according to Eq. (2·29).

Alternatively, if $f_{24} \neq 0$, it is possible to guarantee the alignment of $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{Y}_i^5(M_{\rm cut})$ by requiring that, for every value of $n+m$, the squared bracket in Eq. (7·4) is equal to A_0:

$$B_0 - f_{24} = A_0 = A_1 + f_{24} = A_2 + 2f_{24} = \cdots = A_n + n f_{24}. \tag{7·8}$$

Two parameters are needed to characterize this condition, which we rewrite as:

$$\begin{aligned}
B_0 &= A_0' - 2f_{24}, \\
A_0 &= A_0' - 3f_{24}, \\
A_1 &= A_0' - 4f_{24}, \\
A_2 &= A_0' - 5f_{24}, \\
\cdots &, \\
A_n &= A_0' - (n+3)f_{24}.
\end{aligned} \tag{7·9}$$

The two parameters are f_{24} and A_0, or f_{24} and A_0', and the relation of Eq. (7·7) still holds.

The condition in Eq. (7·6), characterized by only one parameter, A_0, coincide with this in the limit $f_{24} \to 0$. It is easy at this point to see that there is yet another one-parameter condition, which also guarantees the alignment of $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{Y}_i^5(M_{\rm cut})$. This is obtained by setting $A_0' = 0$. In this case, the parameter that fixes the soft couplings in question is f_{24}:

$$B_0 = -2f_{24}, \quad A_0 = -3f_{24}, \quad A_1 = -4f_{24}, \quad \cdots, \quad A_n = -(3+n)f_{24}. \tag{7·10}$$

The expressions for $\boldsymbol{A}_i^5(M_{\rm cut})$ are still those in Eq. (7·7), with $A_0 = -3f_{24}$. It will be shown in Sec. 8 that the two conditions in Eqs. (7·6) and (7·10) correspond to very precise patterns of mediation of SUSY breaking. We anticipate that, in order to realize the condition in Eq. (7·6), the SUSY-breaking field X can only couple to the superpotential, whereas to realize the condition in Eq. (7·10), X couples only to the Kähler potential, in a certain basis. The more general condition in Eq. (7·9) is an interpolation of the other two.

If neither of these conditions is satisfied, and the usual universality for soft couplings is used, in which B_0, A_0, A_1, $A_2, \cdots$ are all independent parameters, the proportionality between $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{Y}_i^5(M_{\rm cut})$ is, in general, broken at $\mathcal{O}(s)$. Thus, sFVs of this order exist at the tree level. We can nevertheless make the effort of assessing the level of arbitrariness introduced in the problem, as we have done in the case of the effective Yukawa couplings. The hope is that, given the drastic reduction of parameters that the usual notion of universality still entails, the constraints acting on the effective trilinear couplings $\boldsymbol{A}_i^5(M_{\rm cut})$ are sufficiently numerous to eliminate a large part of this arbitrariness.

The situation is, indeed, better than it may look at first sight. Since the soft bilinear, trilinear, n-linear couplings of the original NROs are still aligned with the corresponding superpotential couplings, the two sets of effective couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$, $\boldsymbol{Y}_j^{10}(M_{\rm cut}), \cdots$, and $\boldsymbol{A}_i^5(M_{\rm cut})$, $\boldsymbol{A}_j^{10}(M_{\rm cut}), \cdots$ are not completely independent. This is because the matrices that constitute the input parameters in the set of effective Yukawa couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$, $\boldsymbol{Y}_j^{10}(M_{\rm cut}), \cdots$ and in that of effective trilinear couplings are the same up to some coefficients, that is the various A_0, A_1, A_2, $\cdots$, whereas the number of output parameters, the two sets of effective Yukawa and trilinear couplings, is now doubled. Additional constraints with respect to those listed in Secs. 3.1 and 6.1 are therefore expected. In addition to $\mathcal{O}(s\widetilde{m})$- and $\mathcal{O}(s^2\widetilde{m})$-

constraints, there exist also $\mathcal{O}(s^3\widetilde{m})$- and $\mathcal{O}(s^3\widetilde{m}^2)$-constraints, whereas no $\mathcal{O}(s^3)$-constraint exist for the effective Yukawa couplings.

To be specific, in the limit $s \to 0$, we have the three $\mathcal{O}(s)$-constraints for the effective couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$ of Eq. (3·9), and the four $\mathcal{O}(s\widetilde{m})$-constraints for the soft parameters $\boldsymbol{A}_i^5(M_{\rm cut})$, which say trivially that in this limit the soft trilinear couplings are aligned with the Yukawa couplings. The proportionality constant is A_0.

At the next order, there exist five constraints: the $\mathcal{O}(s^2)$-constraint on the effective couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$ of Eq. (3·9), and the following four $\mathcal{O}(s^2\widetilde{m})$-constraints on $\boldsymbol{A}_i^5(M_{\rm cut})$:

$$\begin{aligned}\Big|\Big(&\boldsymbol{A}_i^5(M_{\rm cut}) - (A_1+f_{24})\boldsymbol{Y}_i^5(M_{\rm cut})\\ &-(A_0-A_1-f_{24})\Big(\frac{2}{5}\boldsymbol{Y}_E^5+\frac{3}{5}\boldsymbol{Y}_{DU}^5\Big)(M_{\rm cut})\Big)_{(h,k)}\Big| \le \mathcal{O}(s^2\widetilde{m}),\end{aligned} \tag{7·11}$$

for $i = {\scriptstyle D,E,DU,LQ}$, expressed in terms of A_0 and $A_1 + f_{24}$. Thus, the $\mathcal{O}(s\widetilde{m})$ of the matrices $\boldsymbol{A}_i^5(M_{\rm cut})$ is fixed by these parameters and the effective Yukawa couplings. This means that if only NROs of dimension five are introduced, the matrices $\boldsymbol{A}_i^5(M_{\rm cut})$, and therefore all misalignments between $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{Y}_i^5(M_{\rm cut})$, are known exactly once the couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$ are fixed.

Similar considerations hold also for the couplings $\boldsymbol{A}_j^{10}(M_{\rm cut})$ ($j = {\scriptstyle U,UE,QQ}$), which satisfy the following $\mathcal{O}(s^2\widetilde{m})$-constraints:

$$\begin{aligned}\Big|\Big(&\boldsymbol{A}_j^{10}(M_{\rm cut}) - (A_1+f_{24})\boldsymbol{Y}_j^{10}(M_{\rm cut})\\ &+(A_1-A_0+f_{24})\Big(\frac{2}{5}\left(\boldsymbol{Y}_U^{10}\right)^S+\frac{3}{5}\boldsymbol{Y}_{QQ}^{10}\Big)(M_{\rm cut})\Big)_{(h,k)}\Big| \le \mathcal{O}(s^2\widetilde{m}).\end{aligned} \tag{7·12}$$

The contribution of $\mathcal{O}(s^2\widetilde{m})$ to the matrices $\boldsymbol{A}_i^5(M_{\rm cut})$ and $\boldsymbol{A}_j^{10}(M_{\rm cut})$ is not known, and it is only restricted to satisfy higher-order constraints. In the case of $\boldsymbol{A}_i^5(M_{\rm cut})$, these are

$$\begin{aligned}\Big|\Big(&(\boldsymbol{A}_D^5-\boldsymbol{A}_E^5+\boldsymbol{A}_{LQ}^5-\boldsymbol{A}_{DU}^5)(M_{\rm cut})\\ &-(A_2+2f_{24})\,(\boldsymbol{Y}_D^5-\boldsymbol{Y}_E^5+\boldsymbol{Y}_{LQ}^5-\boldsymbol{Y}_{DU}^5)(M_{\rm cut})\Big)_{(h,k)}\Big| \le \mathcal{O}(s^3\widetilde{m}),\end{aligned} \tag{7·13}$$

a $\mathcal{O}(s^3\widetilde{m})$-constraint, and a constraint at $\mathcal{O}(s^3\widetilde{m}^2)$:

$$\begin{aligned}\Big|\Big(&6\,(A_2-A_0+2f_{24})\left(\boldsymbol{A}_E^5-\boldsymbol{A}_{DU}^5\right)(M_{\rm cut})\\ &-(A_2-A_1+f_{24})\left(2\boldsymbol{A}_E^5+3\boldsymbol{A}_{DU}^5\right)(M_{\rm cut})\\ &-6\,(A_1+f_{24})\,(A_2-A_0+2f_{24})\left(\boldsymbol{Y}_E^5-\boldsymbol{Y}_{DU}^5\right)(M_{\rm cut})\\ &+A_0\,(A_2-A_1+f_{24})\left(2\boldsymbol{Y}_E^5+3\boldsymbol{Y}_{DU}^5\right)(M_{\rm cut})\Big)_{(h,k)}\Big| \le \mathcal{O}(s^3\widetilde{m}^2).\end{aligned} \tag{7·14}$$

It should be reminded here that these constraints, as well as the expressions for the effective trilinear couplings themselves, are obtained in SU(5)-symmetric bases. As mentioned at the end of Sec. 3.3, SU(5)-breaking field redefinitions must be performed to eliminate from the Kähler potential NROs of dimension six in which the fields 24_H acquire *vevs*. These dimension-six operators will be discussed explicitly in Sec. 8. In the basis in which such operators are removed, the $\mathcal{O}(s^3\widetilde{m})$- and $\mathcal{O}(s^3\widetilde{m}^2)$-constraints cannot be used.

This situation is clearly very different from that in which not even the usual universality for soft couplings is required. In this case, the two sets of couplings $\boldsymbol{Y}_i^5(M_{\rm cut})$, $\boldsymbol{Y}_j^{10}(M_{\rm cut}),\cdots$, and $\boldsymbol{A}_i^5(M_{\rm cut})$, $\boldsymbol{A}_j^{10}(M_{\rm cut}),\cdots$ are completely independent. The counting of output parameters (the effective couplings) and of input parameters (the original couplings of the renormalizable and nonrenormalizable operators) is the same for the two sets. The constraints for the effective trilinear couplings $\boldsymbol{A}_i^5(M_{\rm cut})$, $\boldsymbol{A}_j^{10}(M_{\rm cut}),\cdots$ are then formally equal to those for $\boldsymbol{Y}_i^5(M_{\rm cut})$, $\boldsymbol{Y}_j^{10}(M_{\rm cut})\cdots$. (See Sec. 6.1.) It is clearly very difficult to deal with a situation like this. The only hope is that, by expressing these effective couplings in the SU(5)-symmetric basis of Eq. (4·5) as:

$$
\begin{aligned}
\boldsymbol{A}_i^5 &\to (\Delta\widetilde{V}_{5\,i}^\dagger)^T\,\widehat{\boldsymbol{A}}_i^5\,\Delta\widetilde{V}_{10\,i}^\dagger \qquad (i = D,E,DU,LQ),\\
\boldsymbol{A}_j^{10} &\to \left[(\Delta\widetilde{W}_{10\,j}'^\dagger)\widetilde{K}_{\rm CKM}\right]^T\widehat{\boldsymbol{A}}_j^{10}P_{10}\left[\Delta\widetilde{W}_{10\,j}^\dagger\widetilde{K}_{\rm CKM}\right] \qquad (j = U,UE,QQ),
\end{aligned}
\tag{7·15}
$$

it is possible to find some limit in which the number of parameters needed to specify the mismatch matrices in these equations can be reduced considerably, as in the case of the effective Yukawa couplings. Notice that, differently than $\Delta V_{5\,E}$, $\Delta V_{10\,D}$ and $\Delta W_{10\,U}$ (see Eq. (4·6)), the mismatch matrices $\Delta\widetilde{V}_{5\,E}$, $\Delta\widetilde{V}_{10\,D}$ and $\Delta\widetilde{W}_{10\,U}$ do not coincide with the unit matrix. These and all mismatch matrices in these equations, including $\widetilde{K}_{\rm CKM}$, have a tilde to distinguish them from the mismatch matrices for the effective Yukawa couplings. They do reduce to those matrices in the limit $s\to 0$.

7.2. *Effective sfermion masses*

As in the case of trilinear soft couplings, a simple generalization of the concept of universality that assigns the same flavour-independent coupling to operators with the same dimensionality:

$$
d^0_{0,0} = d^0_0, \qquad d^0_{n,m} = d^0_{n+m}, \qquad d^1_{n,m} = d^1_{n+m}, \tag{7·16}
$$

with $n = 0,1,\cdots$, and $m = 1,2,\cdots$, is not sufficient to guarantee the flavour independence of the various contributions to the effective masses.

Already the terms of $\mathcal{O}(s)$ in the expansion for $\widetilde{\boldsymbol{m}}_f^2$ $(f = D^c,L)$ in Eq. (6·15) are, therefore, potentially very dangerous. If flavour dependent, they give (correlated) contributions to sQFVs and sLFVs disentangled from neutrino physics, at the tree level. These contributions appear in the right-right sector of the down squark mass matrix and in the left-left sector of the charged slepton mass matrix, precisely those affected by the large seesaw couplings. Moreover, being of $\mathcal{O}(s)$, they are numerically of the same size of the contributions induced through RGEs by the couplings of the

various seesaw mediators, of $\mathcal{O}(s_{\rm loop})$. The same applies also to the terms of $\mathcal{O}(s^2)$ in the expansion of $\widetilde{\boldsymbol{m}}_f^2$ ($f = {\scriptstyle D^c,L}$) in Eq. (6·15). Although far less harmful than those of $\mathcal{O}(s)$, also these terms can be flavour violating.

Further restrictions on the parameters in Eq. (7·16), such as

$$d_0^0 = d_{n+m}^0 = d_{n+m}^1 = \cdots = \delta, \tag{7·17}$$

could simplify considerably the form of the SUSY-breaking part of the Kähler potential, leading, for example, to the Kähler potential sector:

$$\left[1+\delta f_{\mathsf{X}}\theta^2+\delta^* f_{\mathsf{X}}^*\bar\theta^2+|\delta|^2|f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right]\left\{\bar 5_M\bar 5_M^* + \left[Op^{\mathcal{K}_1}_{\bar 5_M} + Op^{\mathcal{K}_2}_{\bar 5_M} + \text{H.c.}\right]\right\}, \tag{7·18}$$

where we have replaced $\mathsf{X}/M_{\rm cut}$ with $f_{\mathsf{X}}\,\theta^2$.

Clearly, the conditions under which these tree-level sFVs vanish are the conditions under which field redefinitions can remove simultaneously NROs in the SUSY-conserving and SUSY-breaking part of the Kähler potential. As mentioned already, it is sufficient to be able to remove those NROs in the sums of Eqs. (7·18) that contribute within $\mathcal{O}(s^2)$, after the field 24_H is replaced by its *vevs*. (See next section.)

§8. Universal boundary conditions free from sFVs

In order to eliminate sFVs at the tree level, an extension of the conventional notion of universality to the case with nonvanishing NROs must require at least that the field X, which communicates SUSY breaking, couples to operators in the superpotential and Kähler potential in a generation-independent way, and without distinguishing different fields *). In particular, within each of the two potentials, it couples to operators of same dimensionality in the same way.

We refer to the concept of universality outlined above as weak or unrestricted universality, as opposite to that in which the different couplings of X to the various operators in the superpotential and Kähler potential are, in addition, restricted by special conditions. We call this second type of universality restricted or strong universality. Examples of such conditions were outlined already in Secs. 7.1 and 7.2. Here, we reexamine in a systematic way the problem of which conditions are to be imposed to the superpotential and Kähler potential in order eliminate sFVs at the tree level.

In the case of the superpotential, weak universality implies:

$$\begin{aligned} W=&M_{ij}\left(1+bf_X\theta^2\right)\phi_i\phi_j \\ &+Y_{ijk}\left(1+af_X\theta^2\right)\phi_i\phi_j\phi_k+\frac{1}{M_{\rm cut}}C_{ijkl}\left(1+a_1f_X\theta^2\right)\phi_i\phi_j\phi_k\phi_l+\cdots, \end{aligned} \tag{8·1}$$

*) Such a definition excludes the mechanism of gauge mediation of SUSY breaking. It can, however, be generalized to include it, by restricting the above assumption to flavour independence only. In our setup the mediation of SUSY breaking must happen above $M_{\rm GUT}$, at the Planck scale, or quite close to it, if any of the effects discussed in this paper is to be detected. Thus, a component of gauge mediation, if present, has to coexist with that of gravity mediation. This is inevitably present at such scales and, in general, dominant since does not suffer from the loop suppression that plagues the component of gauge mediation. In the following, we restrict our discussion to the case of pure gravity mediation of SUSY breaking.

where the notation is simplified with respect to that of the previous sections, as indices denote both, field types and generations. The fields ϕ_i in this superpotential are SU(5) multiplets, M is M_5 or M_{24}, or a seesaw massive parameter, Y any trilinear coupling, and C any coupling of a NRO of dimension five. The parameter f_X is the usual ratio $F_X/M_{\rm cut}$ that signals the breaking of SUSY, and the quantities b, a, a_1, ... are just numbers. A comparison with the notation of Sec. 7.1 allows the identifications: $B_0 = b f_{\sf X}$, $A_0 = a f_{\sf X}$, $A_1 = a_1 f_{\sf X}$, $\cdots$, and $f_{24} = (b-a) f_{\sf X}$.

We postpone the listing of the general form of what we call the weakly-universal Kähler potential in a noncanonical form, to draw immediately some observations about the behaviour of the superpotential under field redefinitions. These are required to reduce the Kähler potential to be minimal. In general, field redefinitions look like:

$$\phi_i \to \left(c_{ij} + \tilde{c}_{ij} f_X \theta^2\right) \phi_j + \frac{1}{M_{\rm cut}} \left((d_5)_{ijk} + (\tilde{d}_5)_{ijk} f_X \theta^2\right) \phi_j \phi_k + \cdots , \qquad (8\cdot 2)$$

where the second parenthesis, and the following ones, hidden in the dots, are present only when NROs are nonvanishing. (The field $\sf X$, treated here as a spurion, is not included in the counting of the dimensionality of the various operators.) It is obvious that, if the mechanism of mediation of SUSY breaking is even only weakly universal, the couplings $\tilde{c}_{ij}$ and $(\tilde{d}_5)_{ijk}$ are not arbitrary. That is, they cannot depend on field type and generations. Therefore, Eq. (8·2) should be reduced to:

$$\phi_i \to c_{ij} \left(1 + \delta_K f_X \theta^2\right) \phi_j + \frac{1}{M_{\rm cut}} (d_5)_{ijk} \left(1 + \delta_N f_X \theta^2\right) \phi_j \phi_k + \cdots , \qquad (8\cdot 3)$$

where δ_K and δ_N are just numbers.

With these field redefinitions, the above superpotential is modified as:

$$\begin{aligned} W \to\ & M_{lm} c_{li} c_{mj} \left[1 + (b + 2\delta_K) f_X \theta^2\right] \phi_i \phi_j \\ & + Y_{lmn} c_{li} c_{mj} c_{nk} \left[1 + (a + 3\delta_K) f_X \theta^2\right] \phi_i \phi_j \phi_k \\ & + \frac{1}{M_{\rm cut}} 2 M_{lm} c_{li} (d_5)_{mjk} \left[1 + (b + \delta_K + \delta_N) f_X \theta^2\right] \phi_i \phi_j \phi_k \\ & + \frac{1}{M_{\rm cut}} C_{lmno} c_{li} c_{mj} c_{nk} c_{oh} \left[1 + (a_1 + 4\delta_K) f_X \theta^2\right] \phi_i \phi_j \phi_k \phi_h \\ & + \frac{1}{M_{\rm cut}} 3 Y_{lmn} c_{li} c_{mj} (d_5)_{nkh} \left[1 + (a + 2\delta_K + \delta_N) f_X \theta^2\right] \phi_i \phi_j \phi_k \phi_h \\ & + \frac{1}{M_{\rm cut}^2} M_{lm} (d_5)_{lij} (d_5)_{mkh} \left[1 + (b + 2\delta_N) f_X \theta^2\right] \phi_i \phi_j \phi_k \phi_h + \cdots . \end{aligned} \qquad (8\cdot 4)$$

The minimal assumption of Eq. (8·3), however, is not sufficient to preserve the alignment of bilinear, trilinear, n-linear soft terms with the corresponding SUSY-conserving terms, *i.e.* to preserve weak universality, unless:

$$\delta_N = a - b + 2\delta_K = a_1 - a + 2\delta_K = \cdots . \qquad (8\cdot 5)$$

Together with the definition of f_{24}, this implies:

$$a_1 - a = a - b = \cdots = -f_{24}/f_{\sf X}, \qquad (8\cdot 6)$$

which, although derived in a different way, coincides, remarkably, with the condition of Eq. (7·8).

While the stability conditions in Eq. (8·6) relate only SUSY-breaking parameters in the superpotential, those in Eq. (8·5) link SUSY-breaking parameters in the superpotential and in the Kähler potential. These latter conditions indicate that field redefinitions can remove SUSY-breaking operators in the Kähler potential without disturbing the alignment of SUSY-conserving and SUSY-breaking parameters in the superpotential only for specific couplings in both potentials.

Notice that even SUSY-conserving field redefinitions, with $\delta_K = \delta_N = 0$, in general spoil weak universality, except for the particular choice of SUSY-breaking parameters in the superpotential:

$$b = a = a_1 = a_2 = \cdots , \qquad f_{24} = 0. \tag{8·7}$$

This is the first notion of strong universality introduced in the previous section (see Eq. (7·6)). It is encoded in the following particularly simple form for the superpotential:

$$W = \left(1 + a f_X \theta^2\right) \left[M_{ij}\phi_i\phi_j + Y_{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{M_{\rm cut}} C_{ijkl}\,\phi_i\phi_j\phi_k\phi_l + \cdots\right]. \tag{8·8}$$

This type of strong universality is, indeed, stable only if $\delta_K = \delta_N = 0$, that is, if X does not couple to the Kähler potential, which can then be minimized by SUSY-conserving field redefinitions. They modify the superpotential by changing the definition of the parameters M, Y, $C, \cdots$, but leave unchanged the overall parenthesis $(1 + a f_{\mathsf{X}}\theta^2)$, and therefore also the alignment of effective trilinear couplings and effective Yukawa couplings. The soft masses squared are vanishing at $M_{\rm cut}$ and are then generated radiatively, driven by gaugino masses and trilinear couplings.

It would be interesting at this point to check how the superpotential of Eq. (8·8) gets modified when X couples also to the Kähler potential and field redefinitions have nonvanishing δ_K and δ_N.

We postpone momentarily this issue and we consider the other, very restrictive notion of universality for bilinear, trilinear, and n-linear soft parameters of Eq. (7·10), determined also by one parameter only, f_{24}. This can now be expressed as:

$$b : a : a_1 : \cdots = 2 : 3 : 4 : \cdots , \qquad a f_{\mathsf{X}} = -3 f_{24}. \tag{8·9}$$

In this case, the conditions in Eq. (8·5) guarantee that, under field redefinitions, $a f_{\mathsf{X}}$, $b f_{\mathsf{X}}$, $a_1 f_{\mathsf{X}}$ are shifted in such a way to leave the ratios $b : a : a_1 \cdots$ unchanged. From Eq. (8·4) it is easy to see that $a f_{\mathsf{X}}$ becomes:

$$a f_{\mathsf{X}} = -3 f_{24} \to -3\left(f_{24} - \delta_K f_{\mathsf{X}}\right). \tag{8·10}$$

The superpotential, which has the form:

$$\begin{aligned} W = {} & M_{ij}\left(1 - 2 f_{24}\theta^2\right)\phi_i\phi_j \\ & + Y_{ijk}\left(1 - 3 f_{24}\theta^2\right)\phi_i\phi_j\phi_k + \frac{1}{M_{\rm cut}} C_{ijkl}\left(1 - 4 f_{24}\theta^2\right)\phi_i\phi_j\phi_k\phi_l + \cdots , \end{aligned} \tag{8·11}$$

before field redefinitions, remains of the same type, except for a modification of the parameters M, Y, $C, \cdots$ and for the shift of f_{24}:

$$f_{24} \to (f_{24} - \delta_K f_{\mathsf{X}})\,. \tag{8·12}$$

The alignment $\boldsymbol{A}_i^5 = A_0 \boldsymbol{Y}_i^5$ of Eq. (7·7) is satisfied for $A_0 = -3f_{24}$, with different values of f_{24} before and after field redefinitions. Notice that, if $\delta_K f_{\mathsf{X}} = f_{24}$, it is possible to find a basis in which the shifted value of f_{24}, and therefore all the parameters b, a, a_1, $\cdots$, are vanishing. In this basis, X couples only to the Kähler potential. Bilinear, trilinear and n-linear soft terms satisfying the condition of Eq. (8·9) are generated as a result of field redefinitions.

This fact illustrates why the superpotential in Eq. (8·8) tends to be destabilized, unless X is decoupled from the Kähler potential. It is simply due to the fact that SUSY-breaking field redefinitions generate f_{24} even if this vanishes in one basis.

It seems therefore plausible that, as the previous one, also the more general notion of universality of Eq. (7·9), is stable under the field redefinitions of Eq. (8·3). Depending on the two parameters A_0' and f_{24}, this is nothing else but an interpolation of the two notions considered above. It can now be expressed as:

$$b - a' : a - a' : a_1 - a' \cdots = 2 : 3 : 4 \cdots\,, \qquad (a - a') f_{\mathsf{X}} = -3 f_{24}, \tag{8·13}$$

with $a' = A_0'/f_{\mathsf{X}}$. By rewriting the parameters b, a, a_1, $\cdots$ as $a' + (b - a')$, $a' + (a - a')$, $a' + (a_1 - a')$, $\cdots$, we obtain the following form for Eq. (8·5):

$$\delta_N = \big[(a - a') - (b - a')\big] + 2\delta_K = \big[(a_1 - a') - (a - a')\big] + 2\delta_K. \tag{8·14}$$

These conditions guarantee that the ratios $b - a' : a - a' : a_1 - a' \cdots$ are not modified by field redefinitions, which shift $(a - a') f_{\mathsf{X}}$ as follows:

$$(a - a') f_{\mathsf{X}} = -3 f_{24} \to -3\,(f_{24} - \delta_K f_{\mathsf{X}})\,. \tag{8·15}$$

Thus, the superpotential, which has the interesting form:

$$W = \left(1 + a' f_{\mathsf{X}} \theta^2\right) \Big[M_{ij} \left(1 - 2 f_{24} \theta^2\right) \phi_i \phi_j + Y_{ijk} \left(1 - 3 f_{24} \theta^2\right) \phi_i \phi_j \phi_k$$
$$+ \frac{1}{M_{\rm cut}} C_{ijkl} \left(1 - 4 f_{24} \theta^2\right) \phi_i \phi_j \phi_k \phi_l + \cdots \Big] \tag{8·16}$$

remains formally unchanged by field redefinitions, except for the shift of f_{24} of Eq. (8·15), and a modification of the parameters M, Y, $C, \cdots$. The alignment $\boldsymbol{A}_i^5 = A_0 \boldsymbol{Y}_i^5$ of Eq. (7·7) is again satisfied for $A_0 = A_0' - 3f_{24}$, with different values of f_{24} before and after field redefinitions.

It is to the form in Eq.(8·16) that the superpotential in Eq. (8·8) is brought by field redefinitions with $\delta_K f_{\mathsf{X}} = -f_{24}$. Vice versa, field redefinitions with $\delta_K f_{\mathsf{X}} = f_{24}$ bring the superpotential Eq.(8·16) to that in Eq. (8·8).

We have discovered so far that a weakly-universal superpotential with NROs as in Eq. (8·1) is unstable under field redefinitions, even SUSY-conserving ones, if NROs exist also in the Kähler potential. There would be no problem at all if such

a superpotential could be somehow realized in a basis in which the Kähler potential is already canonical. In general, however, this is not the case and the superpotential must be strongly universal to prevent that field redefinitions spoil even its weak universality. Any of the two forms in Eq. (8·8) or (8·16) is equally suitable.

Notice that in the MSSM limit, *i.e.* if only light superfields are present, NROs are negligible in both, the superpotential and the Kähler potential, and the need for a restricted notion of universality to avoid sFVs at the tree level does not exist.

A weekly-universal Kähler potential, to which Eq. (7·16) already alludes, is

$$
\begin{aligned}
K(\mathsf{X}^*,\mathsf{X}) = & \Big[1 - \alpha_4 f_{\mathsf{X}}\theta^2 - \alpha_4^* f_{\mathsf{X}}^*\bar{\theta}^2 + \Big(|\alpha_4|^2 - d^2\Big)\,|f_{\mathsf{X}}|^2\,\theta^2\bar{\theta}^2\Big]\,\phi_i^*\phi_i \\
& - \Big\{\frac{(d_5)_{ijk}}{M_{\rm cut}}\left[1 + \alpha_5^0 f_{\mathsf{X}}\theta^2 + \beta_5^{0*} f_{\mathsf{X}}^*\bar{\theta}^2 + \gamma_5^0\,|f_{\mathsf{X}}|^2\,\theta^2\bar{\theta}^2\right]\phi_i^*\phi_j\phi_k + \text{H.c.}\Big\} \\
& - \frac{(d_6)_{ijkh}}{M_{\rm cut}^2}\left[1 + \alpha_6^0 f_{\mathsf{X}}\theta^2 + \alpha_6^{0*} f_{\mathsf{X}}^*\bar{\theta}^2 + \gamma_6^0\,|f_{\mathsf{X}}|^2\,\theta^2\bar{\theta}^2\right]\phi_i^*\phi_j^*\phi_k\phi_h \\
& - \Big\{\frac{(d_6')_{ijkh}}{M_{\rm cut}^2}\left[1 + \alpha_6' f_{\mathsf{X}}\theta^2 + \beta_6'^* f_{\mathsf{X}}^*\bar{\theta}^2 + \gamma_6'\,|f_{\mathsf{X}}|^2\,\theta^2\bar{\theta}^2\right]\phi_i^*\phi_j\phi_k\phi_h + \text{H.c.}\Big\},
\end{aligned}
\tag{8·17}
$$

where, without loss of generality, we have already assumed the canonical form for the SUSY-conserving dimension-four operators. The parameter d^2 is assumed to be real and positive, since, as will be shown later, it determines the soft mass squared of the fields ϕ_i. The couplings $(d_5)_{ijk}$ and $(d_6')_{ijkl}$ are symmetric under permutations of the last two and the last three indices, respectively, and d_6 satisfies the following relations: $(d_6)_{ijkh} = (d_6)_{jikh} = (d_6)_{ijhk} = (d_6^*)_{khij}$. For simplicity we also limit this discussion to operators up to dimension six. As will be shown in Sec. 10, within the precision of our calculation, these are sufficient for the determination of the boundary conditions for effective soft masses.

As already discovered in the case of weakly-universal superpotentials, also this Kähler potential is unstable under field redefinitions. It is a simple exercise to show this. As in the case of the superpotential, the stability of the weak universality implies conditions among its various parameters and the field-redefinition parameters, which we do not report here explicitly. It is straightforward, although tedious, to show that for a Kähler potential and a superpotential in the basis specified by Eqs. (8·17) and (8·8), constraints on the various parameters in $K(\mathsf{X}^*,\mathsf{X})$ are induced by the requirement that NROs of dimension five can be eliminated. These constraints, together with the conditions of stability of the weak universality of both potentials, allow to rewrite $K(\mathsf{X}^*,\mathsf{X})$ in the following factorizable form:

$$
K(\mathsf{X}^*,\mathsf{X}) = \Big[1 - \delta_K f_{\mathsf{X}}\theta^2 - \delta_K^* f_{\mathsf{X}}^*\bar{\theta}^2 + \big(|\delta_K|^2 - d^2\big)\,|f_{\mathsf{X}}|^2\theta^2\bar{\theta}^2\Big]K,
\tag{8·18}
$$

where $\delta_K = \alpha_4$, and K is the SUSY-conserving part of the Kähler potential:

$$
K = \phi_i^*\phi_i - \left[\frac{(d_5)_{ijk}}{M_{\rm cut}}\phi_i^*\phi_j\phi_k + \text{H.c.}\right] - \frac{(d_6)_{ijkh}}{M_{\rm cut}^2}\phi_i^*\phi_j^*\phi_k\phi_h
$$

$$-\left[\frac{(d_6')_{ijkh}}{M_{\rm cut}^2}\phi_i^*\phi_j\phi_k\phi_h + {\rm H.c.}\right]. \tag{8·19}$$

The form in Eq. (8·18) is one example of a Kähler potential with strong universality. The superpotential is still that of Eq. (8·8), and f_{24} vanishes. *) Notice that the guess made in Sec. 7.2 (see Eq. (7·18)) for a Kähler potential, which would not lead to sFVs at the tree-level, is in the right direction but not completely correct. The expression in Eq. (7·18) is, indeed, very similar to that in Eq. (8·18), when δ_K is identified with $-\delta$, but, in reality, less general, as it has one parameter less.

It is interesting to see that the set of the superpotential (8·8) and the Kähler potential (8·18) gives the same scalar potential as supergravity model with sequestered superpotential and the Kähler potential where each potential for the visible sector is given by Eqs. (8·8) and (8·18) with $\mathsf{X}\to 0$, if we set $d^2 = |\delta_K|^2$.

Thanks to the factorization of Eq. (8·18), the effort in bringing $K(\mathsf{X}^*,\mathsf{X})$ to a form as close as possible to the canonical one, reduces to that of minimizing K. The elimination in K of the dimension-five NROs, which is possible to achieve, automatically eliminates also all the corresponding SUSY-breaking NROs of the same dimensionality. In the language of the previous sections: since it is possible to redefine away the dimension-five NROs in $Op^{\mathcal{K}_1}_{\bar 5_M}$, also all the dimension-five NROs in $Op^{\mathcal{K}_1}_{\bar 5_M}(\mathsf{X})$, $Op^{\mathcal{K}_1}_{\bar 5_M}(\mathsf{X}^*)$, and $Op^{\mathcal{K}_1}_{\bar 5_M}(\mathsf{X}^*,\mathsf{X})$ can be eliminated. This is particularly important as it enables the elimination of the most dangerous arbitrary sFVs in the boundary values for soft masses, *i.e.* those which can be of the same size as the sFVs induced radiatively by the neutrino seesaw couplings.

In fact, the situation is even better than that just described. Both terms in square brackets in Eq. (8·19) can be removed through field redefinitions, as they both have all fields but one with the same chirality, therefore reducing $K(\mathsf{X}^*,\mathsf{X})$ to:

$$K(\mathsf{X}^*,\mathsf{X}) = \left[1-\delta_K f_{\mathsf{X}}\theta^2 - \delta_K^* f_{\mathsf{X}}^*\bar\theta^2 + \left(|\delta_K|^2 - d^2\right)|\,f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right] \times\left\{\phi_i^*\phi_i - \frac{(d_6)_{ijkh}}{M_{\rm cut}^2}\phi_i^*\phi_j^*\phi_k\phi_h\right\}. \tag{8·20}$$

While these field redefinitions are SUSY conserving, SUSY-breaking ones are needed to bring $K(\mathsf{X}^*,\mathsf{X})$ to the even simpler form:

$$\begin{aligned}K(\mathsf{X}^*,\mathsf{X}) &= \left[1-d^2|f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right]\phi_i^*\phi_i \\ &+ \left[1+\delta_K f_{\mathsf{X}}\theta^2 + \delta_K^* f_{\mathsf{X}}^*\bar\theta^2 + \left(|\delta_K|^2 - d^2\right)|\,f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right]\frac{(-d_6)_{ijkh}}{M_{\rm cut}^2}\phi_i^*\phi_j^*\phi_k\phi_h,\end{aligned} \tag{8·21}$$

*) If in the basis of Eq. (8·17) for $K(\mathsf{X}^*,\mathsf{X})$ the superpotential is that of Eq. (8·16), the same procedure followed in the previous case yields a slightly more complicated form for the Kähler potential than that in Eq. (8·18), but equivalent to it. Field redefinitions, indeed, bring it to the factorizable form of Eq. (8·18), with $\delta_K f_{\mathsf{X}} = \alpha_4 f_{\mathsf{X}} - f_{24}$, while the superpotential is brought to the form in Eq. (8·8). Thus, irrespective of whether the superpotential is in the form of Eq. (8·8) or (8·16) in the basis of Eq. (8·17) for the Kähler potential, the final form of the superpotential in the basis in which the Kähler potential is factorized as in Eq. (8·18) is always that of Eq. (8·8), *i.e.* also a factorized one.

in which the form of the dimension-four operators is canonical. Through them, f_{24} is generated again, with the value $f_{24} = -\delta_K f_{\mathsf{X}}$, and the superpotential is brought from the unstable form of Eq. (8·8) to the more general form of Eq. (8·16). It is clear at this point why the guess of Eq. (7·18) for the Kähler potential is not correct. These SUSY-breaking field redefinitions would reduce the soft masses to be vanishing.

For all practical purposes, we can neglect all dimension-six NROs, except when they contain two 24_H fields, with opposite chirality, as these fields can acquire *vevs*, thus contributing in general in a flavour dependent way to soft masses. If these two fields are $\phi_j^*\phi_k$, then the term multiplied by the square bracket in the second line of Eq. (8·21) becomes:

$$s^2(-d_6)_{ijjh}\left(1 + f_{24}\theta^2 + f_{24}^*\bar\theta^2 + |f_{24}|^2\theta^2\bar\theta^2\right)\phi_i^*\phi_h, \tag{8·22}$$

and the index j labels the components of the field 24_H. Group theoretical factors are suppressed in this expression and are assumed to be reabsorbed by d_6. The second line of Eq. (8·21) is

$$\left[1 + \Big(\delta_K f_{\mathsf{X}} + f_{24}\Big)\theta^2 + \Big(\delta_K^* f_{\mathsf{X}}^* + f_{24}^*\Big)\bar\theta^2 + \Big(|\delta_K f_{\mathsf{X}} + f_{24}|^2 - d^2|f_{\mathsf{X}}|^2\Big)\theta^2\bar\theta^2\right]$$
$$\times\, s^2(-d_6)_{ijjh}\phi_i^*\phi_h, \tag{8·23}$$

and recalling that $f_{24} = -\delta_K f_{\mathsf{X}}$, the Kähler potential of Eq. (8·21) reduces to:

$$K(\mathsf{X}^*,\mathsf{X}) \simeq \left[1 - d^2|f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right]\left(\delta_{ih} - s^2(d_6)_{ijjh}\right)\phi_i^*\phi_h, \tag{8·24}$$

up to negligible NROs of dimension six. There is something very special about this cancellation, which rests on the simultaneous presence of strong universality in both potentials. Had it been $\delta_K f_{\mathsf{X}} + f_{24} \neq 0$, SU(5)-breaking field redefinitions could have removed the two terms proportional to θ^2 and $\bar\theta^2$, but not the term proportional to $\theta^2\bar\theta^2$, which multiplies d_6. These field redefinitions would have also shifted f_{24} of a quantity of $\mathcal{O}(s^2)$ proportional to d_6. Hence, misalignments would have been introduced in the superpotential, correlated to the residual sFVs of $\mathcal{O}(s^2)$ in the Kähler potential.

A further SUSY-conserving, but SU(5)-breaking field redefinition, yields:

$$K(\mathsf{X}^*,\mathsf{X}) \simeq \left[1 - d^2|f_{\mathsf{X}}|^2\theta^2\bar\theta^2\right]\phi_i^*\phi_i. \tag{8·25}$$

Notice that these are also flavour-violating field redefinitions of $\mathcal{O}(s^2)$. Since they are SUSY-conserving, their effect is only that of modifying the parameters M, Y, $C, \cdots$ in the superpotential, that is, of modifying simultaneously supersymmetric parameters and corresponding soft parameters, without introducing spurious misalignments at the tree level *). The superpotential is still formally that of Eq. (8·16), as if these field redefinitions had not taken place. They can, therefore, be totally ignored if one needs to use this superpotential in an SU(5)-symmetric basis. Three

*) Possible misalignments may be introduced at the loop level, which however would be of $\mathcal{O}(s^2 \times s_{\rm loop})$ and, therefore, completely negligible for our purposes.

free parameters, together with the gaugino mass, are needed in this case to specify all soft terms:

$$a', \quad f_{24}, \quad d^2, \tag{8·26}$$

with d^2 real and positive. Had we taken $d^2 = |\alpha_4|^2$ in Eq. (8·17), Eq. (8·25) would now be:

$$K(\mathsf{X}^*, \mathsf{X}) \simeq \Big[1 - |f_{24}|^2 \theta^2 \bar{\theta}^2\Big] \phi_i^* \phi_i, \tag{8·27}$$

and only the parameters a' and f_{24} would have to be specified in addition to the gaugino mass.

Summarizing, for a SUSY-breaking mediator coupled to operators of both potentials, the condition to avoid sFVs at the tree level is that of having, in the same basis, the superpotential of Eq. (8·8) and the Kähler potential of Eq. (8·18), or equivalently, in the form of Eq. (8·16) and (8·25), respectively.

As mentioned earlier, it is possible that the SUSY-breaking mediator couples only to the superpotential. The Kähler potential, now SUSY conserving, is in this case that of Eq. (8·19). This can be easily reduced with SUSY-conserving field redefinitions to have the form of the potential in curly brackets in Eq. (8·20). This possibility was brought to attention when discussing the superpotential in Eq. (8·8). Since in this basis $f_{24} = 0$, the Kähler potential reduces directly to:

$$K \simeq \Big[\delta_{ih} - s^2 (d_6)_{ijjh}\Big] \phi_i^* \phi_h, \tag{8·28}$$

when the fields $\phi_j^* \phi_j$, identified with $24_H^* 24_H$, acquire their scalar *vev*. Again, an SU(5)-breaking field redefinition can remove the second term in parentheses.

Notice that, if the condition of universality for the superpotential is relaxed to be weak instead of strong in the basis of Eq. (8·17) for the Kähler potential, it is still possible to tune the Kähler potential to have the form in Eq. (8·18). An attempt to minimize it in this case yields:

$$K(\mathsf{X}^*, \mathsf{X}) \simeq \Big[\delta_{ih} + \Big(|d|^2 |f_{\mathsf{X}}|^2 \delta_{ih} - s^2 (d_6)_{ijjh} |f_{24} + \delta_K f_{\mathsf{X}}|^2\Big) \theta^2 \bar{\theta}^2\Big] \phi_i^* \phi_h, \tag{8·29}$$

up to negligible dimension-six NROs, but destroys also the weak universality of the superpotential. Thus, situations in which the superpotential is weakly-universal and the Kähler potential is canonical, up to terms of $\mathcal{O}(s^2)$, which are arbitrary, and in general flavour- and CP-violating, can only be achieved at the expense of some severe tuning. Highly tuned is also a situation in which the superpotential is weakly-universal and the Kähler potential is already canonical, as in Eq. (8·25).

§9. Emerging scenarios

The analysis of the previous sections shows clearly that the seesaw-induced predictions for sFVs in an MSSU(5) setting can vary considerably. There are of course the interesting differences induced by the various types of seesaw mechanism. If large enough, they may help distinguish which of the three types of seesaw mediators gives rise to the low-energy dimension-five neutrino operator of Eq. (2·39).

There are also the worrisome modifications that the seesaw-induced sFVs may undergo, depending on the type of NROs introduced and the role that they play. The arbitrariness of NROs is the key ingredient exploited to cure the two major problems of the MSSU(5) model, the unacceptable predictions for the values of fermion masses and for the proton-decay rate. How to use it to correct the fermion spectrum is straightforward and well known, whereas the recipe to be followed to suppress the decay rate of the proton is not simple and presumably not unique. Indeed, we have argued in Sec. 5 that baryon-number violating NROs may also produce a sufficient suppression of this rate, in addition to that due to the freedom in the effective Yukawa couplings of matter fields to colored Higgs triplets.[43)–45)] Thus, it is not yet clear whether the constraints from proton decay imply that the mismatch matrices $\Delta V_{5\,i}$, $\Delta V_{10\,i}$ ($i = D,E,DU,LQ$), and $\Delta W_{10\,j}$, $\Delta W'_{10\,j}$ ($j = U,UE,QQ$) discussed in Sec. 4 are trivial or not. Waiting to get a definite answer on this issue, we assume here that these matrices are nontrivial, *i.e.* that they are substantially different from the unit matrix.

As for the arbitrariness that NROs can induce in the boundary conditions for soft parameters, the results obtained in Sec. 8 can be summarized as follows. In general, this arbitrariness causes particularly dangerous tree-level sFVs already at $\mathcal{O}(s)$ that completely obscure the effect of the large seesaw Yukawa couplings. There exist however special field- and flavour-independent couplings of the SUSY-breaking mediator field X to the superpotential and the Kähler potential, or strongly-universal couplings, which are not destabilized by field redefinitions, and which allow to eliminate such tree-level sFVs. In addition to the gaugino mass, $M_{1/2}$, then, only three parameters are sufficient to specify all effective soft couplings:

$$\widetilde{m}_0^2, \quad A'_0, \quad f_{24}, \tag{9·1}$$

the first two of which are expressed in Sec. 8 as $d^2|f_{\mathsf{X}}|^2$ and $a' f_{\mathsf{X}}$. At $M_{\rm cut}$, all effective soft masses squared are equal to $\widetilde{m}_0^2$:

$$\widetilde{\boldsymbol{m}}_i^2(M_{\rm cut}) = \widetilde{m}_0^2\,\mathbf{1}, \tag{9·2}$$

for any field i. The effective trilinear couplings are aligned with the effective Yukawa couplings, as for example in:

$$\boldsymbol{A}_i^5(M_{\rm cut}) = A_0 \boldsymbol{Y}_i^5 \qquad (i = D,E,DU,LQ), \tag{9·3}$$

with

$$A_0 = A'_0 - 3 f_{24}. \tag{9·4}$$

All B parameters are expressed in terms of the corresponding superpotential massive parameter as [*)]:

$$\boldsymbol{B}_i(M_{\rm cut}) = B_0 M_i, \tag{9·5}$$

[*)] We have not given explicitly definitions of effective bilinear couplings in terms of the original renormalizable and nonrenormalizable operators. They are completely analogous to those for the trilinear effective couplings.

with:

$$B_0 = A_0' - 2f_{24} = A_0 + f_{24}. \tag{9·6}$$

That is, we recover for effective couplings the type of universality usually advocated for models without NROs. The choice of three parameters at $M_{\rm cut}$, can indeed be the much more familiar:

$$\widetilde{m}_0^2, \quad A_0, \quad B_0, \tag{9·7}$$

with f_{24} fixed in terms of B_0 and A_0: $f_{24} = B_0 - A_0$. The choice $d^2 = |\alpha_4|^2$ in Eq. (8·26) corresponds to fixing B_0 among the above parameters to be $(B_0 - A_0)^2 = \widetilde{m}_0^2$, reproducing the minimal supergravity result.[15] Notice that all the bilinear terms discussed above are for the superheavy fields, and the MSSM B-term can be arbitrary even in this case, depending on the fine-tuning for the μ term, as shown in Sec. 2.2.

The case discussed so far is a somewhat special case, which rests uniquely on the requirement of simultaneous strong universality in the superpotential and in the Kähler potential. With this choice of universality, the arbitrariness induced by NROs remains confined to the Yukawa sector. An estimate of how the predictions of the seesaw-induced sFVs get modified by the mismatch matrices will be given later in this section.

Beyond this special case, the situation becomes considerably more complicated. Following Sec. 8, we know that by relaxing the condition of universality for the superpotential to be weak instead of strong in the basis of Eq. (8·17) for the Kähler potential, we can tune the Kähler potential to be as in Eq. (8·18), which leads to the form in Eq. (8·29), by field redefinitions. That is, effective soft masses squared are universal at $\mathcal{O}(1)$, but arbitrary at $\mathcal{O}(s^2)$. In addition, the field redefinitions through which such form of the Kähler potential is obtained, spoil also the weak universality of the superpotential at $\mathcal{O}(s)$. At this order, the effective trilinear couplings, not aligned anymore to the effective Yukawa couplings, are arbitrary 3×3 matrices. It is possible to parametrize them as shown in Eq. (7·15), in terms of diagonal matrices and new unitary matrices of diagonalization mismatch, $\Delta\widetilde{V}_{5\,i}$, $\Delta\widetilde{V}_{10\,i}$ ($i = {\scriptstyle D,E,DU,LQ}$), $\Delta\widetilde{W}_{10\,j}$, $\Delta\widetilde{W}'_{10\,j}$ ($j = {\scriptstyle U,UE,QQ}$), and $\widetilde{K}_{\rm CKM}$. The number of free parameters, with which one has to deal, is, however, overwhelmingly large.

As explained in Sec. 8, it is also possible to envisage another tuned situation, in which the superpotential has weak universality in the basis of Eq. (8·25) for the Kähler potential. The effective soft masses squared are in this case universal. Superpotential couplings and corresponding soft couplings are in general aligned, but the alignment is lost at $\mathcal{O}(s)$ for effective Yukawa couplings and effective trilinear couplings. This is the scenario analyzed in Ref. 19). It is considerably simpler than the one discussed earlier, because, although not aligned to the effective Yukawa couplings, the effective trilinear couplings depend on them in precise ways, at least at $\mathcal{O}(s)$. In fact, the constraint in Eq. (7·11) shows that the misalignment between, for example, $\boldsymbol{A}_i^5$ and the corresponding $\boldsymbol{Y}_i^5$ is given in terms of various effective Yukawa

couplings and a new parameter, A_1, in addition to A_0 and f_{24}:

$$\boldsymbol{A}_i^5(M_{\rm cut}) = \left\{(A_1 + f_{24}) + \frac{A_0 - A_1 - f_{24}}{5}\left[\frac{2\boldsymbol{Y}_E^5 + 3\boldsymbol{Y}_{DU}^5}{\boldsymbol{Y}_i^5}\right](M_{\rm cut})\right\}\boldsymbol{Y}_i^5(M_{\rm cut}). \tag{9·8}$$

Thus, at this order, all effective soft couplings can be specified in terms of:

$$\widetilde{m}_0^2, \quad A_0, \quad A_1, \quad f_{24}, \tag{9·9}$$

and the gaugino mass, $M_{1/2}$. The remaining arbitrariness in the expressions of the various effective trilinear couplings is, at this order, the arbitrariness of the effective Yukawa couplings, which are not fixed by their $\mathcal{O}(s^2)$-constraints given in Eq. (3·9). Constraints at $\mathcal{O}(s^2)$ among the various effective trilinear couplings do exist. These are the $\mathcal{O}(s^3\widetilde{m})$- and the $\mathcal{O}(s^3\widetilde{m}^2)$-constraints given explicitly for the couplings $\boldsymbol{A}_i^5$ ($i = {\scriptstyle D,E,DU,LQ}$), in Eqs. (7·13) and (7·14). They link effective trilinear couplings and effective Yukawa couplings at this order making also use of the parameters A_0, A_1, f_{24}, and an additional one, A_2. Although insufficient to fix the effective trilinear couplings completely, they seem nevertheless useful to limit their arbitrariness.

§10. Picture of effective couplings at the quantum level

The picture of effective couplings outlined so far, may be valid at the quantum level under two obvious conditions: i) the dynamical part of the field 24_H that appears in NROs can be neglected when considering low-energy physics; ii) NROs that do not contain the field 24_H are also irrelevant for low energy, except perhaps for proton decay and cosmology.[61] These two conditions are not valid in general, but only for a limited accuracy of our calculation of sFVs. It can be easily shown that, within this accuracy, the effective Yukawa couplings evolve in the same way as the couplings of an MSSU(5) model with the SU(5) symmetry broken at $\mathcal{O}(s)$ everywhere except than in the gauge sector. Key ingredient for this proof is the running of the *vevs* of the adjoint Higgs field 24_H, discussed also in Sec. 2.2 and Appendix C.

10.1. *Effective-picture validity and allowed accuracy*

It is easy to see that the conditions i) and ii) listed above are in general violated. We show it here in the case of effective Yukawa couplings, but the same discussion can be applied to the holomorphic SUSY-breaking terms. It is shown in Fig. 2 how two operators in the superpotential, say for definiteness the NRO in $Op^5|_5$ with coefficient $C^5_{0,1}$ and the renormalizable one $\lambda_5 5_H 24_H \bar{5}_H$ induce in the Kähler potential the NRO:

$$x\lambda_5\, 5_H^* 10_M\, C^{5\,T}_{0,1}\, \bar{5}_M \qquad \Big(x \propto \frac{s_{\rm loop}}{M_{\rm cut}}\Big). \tag{10·1}$$

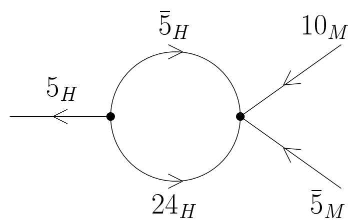

Fig. 2. NRO induced by Op^5 and $\lambda_5 5_H 24_H \bar{5}_H$ in $W_H^{\rm MSSU(5)}$.

The field redefinition needed to reabsorb it:

$$5_H \ \rightarrow\ 5_H - x\lambda_5 10_M\, C^{5\,T}_{0,1}\, \bar{5}_M, \tag{10·2}$$

shifts also the various terms in the superpotential to induce NROs such as:

$$-\, x\lambda_5\, 10_M\, Y^{10}\, 10_M 10_M\, C^{5\,T}_{0,1}\, \bar{5}_M, \qquad -x\lambda_5\, N^c\, Y^{\rm I}_N\, \bar{5}_M 10_M\, C^{5\,T}_{0,1}\, \bar{5}_M, \tag{10·3}$$

which do not contain the field 24_H and are, therefore, not included in the linear combinations that define the various $\boldsymbol{Y}^5_i$, as well as:

$$-\, x\lambda_5 \left(M_5 \bar{5}_M\, C^5_{0,1}\, 10_M \bar{5}_H + \lambda_5\, \bar{5}_M\, C^5_{0,1}\, 10_M 24_H \bar{5}_H \right). \tag{10·4}$$

Thus, at the one-loop level, the dynamical part of the field 24_H can give rise to corrections to some of the coefficients included in the effective couplings $\boldsymbol{Y}^5_i$, although not to the complete linear combinations that define them. In our specific case, for example, it induces a correction to the coefficient of the renormalizable operator $Op^5|_4$ of $\mathcal{O}(s \times s_{\rm loop})$, since M_5 is of the same order of v_{24}, and one to the coefficient of $Op^5|_5$ itself, of $\mathcal{O}(s_{\rm loop})$ and therefore of $\mathcal{O}(s \times s_{\rm loop})$ to the effective couplings $\boldsymbol{Y}^5_i$.

Similarly, $Op^5|_4$ and a NRO like the first in Eq. (10·3) not containing the field 24_H, for example the dimension-five operators in the class $Op^{\rm PD}$ in Eq. (3·19), give also rise at the one-loop level to the Kähler potential NRO $5^*_H 10_M \bar{5}_M$ and therefore superpotential NROs such as those listed above. (See Fig. 3.) The suppression factor is also in this case $x \propto s_{\rm loop}/M_{\rm cut}$ if the dimension-five NRO in $Op^{\rm PD}$ exists at the tree level and the various components of its coefficient $C^{\rm PD}_0$ are of $\mathcal{O}(1)$. Strictly speaking, some of the flavour component of this coefficient must be sufficiently suppressed to avoid conflict with the experimental lower bound on the lifetime of the proton. Others, however, may happen to be unsuppressed.

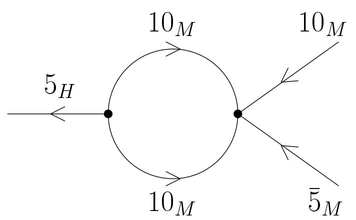

Fig. 3. NRO induced by $Op^{\rm PD}$ and $10_M 10_M 5_H$ in $W_M^{\rm MSSU(5)}$.

The same apparently-problematic results are also induced, for example, by the tree-level NRO $N^c \bar{5}_M 10_M \bar{5}_M$ and the seesaw operator $-N^c Y^{\rm I}_N \bar{5}_M 5_H$.

A closer inspection shows, however, that the only corrections of $\mathcal{O}(s \times s_{\rm loop})$ are those to terms of $\boldsymbol{Y}^5_{DU}$ and $\boldsymbol{Y}^5_{LQ}$, *i.e.* effective couplings of Yukawa interactions involving the superheavy Higgs triplet H^C_D. Through RGEs, these influence effective soft masses squared and trilinear couplings, and can therefore induce sFVs at $\mathcal{O}(s \times s^2_{\rm loop})$. In contrast, because the combination $\mu_2 = M_5 - (1/2)\sqrt{6/5}\lambda_5 v_{24}$ has already been tuned to be of $\mathcal{O}(M_{\rm weak})$ (see Sec. 2.2), the corrections that the two operators in Eq. (10·4) bring to the effective couplings of the Yukawa interactions that involve light Higgs fields, $\boldsymbol{Y}^5_D$ and $\boldsymbol{Y}^5_E$ are negligibly small, of $\mathcal{O}((M_{\rm weak}/M_{\rm cut}) \times s_{\rm loop})$.

Thus, the picture of effective couplings can be retained at the quantum level if we limit the precision of our calculation of sFVs, to be at most of $\mathcal{O}(s \times s_{\rm loop})$, but it cannot be used for other quantities, such as the proton-decay Wilson coefficients, which depend at the tree-level on the Yukawa couplings of the superheavy fields. It

will be shown in the next subsection, and proven explicitly in Appendix D, that, up to a precision of $\mathcal{O}(s \times s_{\rm loop})$ for sFVs, the effective couplings behave as couplings of renormalizable operators, *i.e.* they obey RGEs of renormalizable operators.

This limitation in precision is certainly not a problem, since it does not seem possible to probe sFVs beyond $\mathcal{O}(s \times s_{\rm loop}) = 10^{-4}$. If a higher sensitivity could, hypothetically, be achieved, the picture of effective coupling would have to be replaced by a different treatment of the NROs, or, possibly, would require an enlargement of basis, to contain also effective couplings for the NROs themselves.

The precision $\mathcal{O}(s \times s_{\rm loop})$ is that induced at the one-loop level by NROs of dimension five, that is $Op^5|_5$ and $Op^{10}|_5$ in the Yukawa sector, with a nondynamical 24_H. Thus, our calculation of low-energy quantities is not sensitive to quantum contributions from higher-dimension NROs, as well as to resummation effects obtained by solving the RGEs for effective couplings. We nevertheless retain this picture, because we find it physically very clear. It leads to straightforward matching conditions at $M_{\rm GUT}$ and to a transparent interpretation of the boundary conditions at the cutoff scale.

There is one effect of this order that we do not include. As mentioned already, we neglect NROs in the Higgs sector. Barring small values of the coupling λ_{24}, the effect of these NROs is that of producing small shifts in the mass of the superheavy fields, such as the colored Higgs triplets. Shifts in the mass of these fields result in a modification of the scale at which these fields decouple from light matter. On the product of effective couplings that enter in the evaluation of sLFVs:

$$\frac{1}{16\pi^2}\boldsymbol{Y_N^{\rm I}}^\dagger\boldsymbol{Y_N^{\rm I}} \ln\left(\frac{M_{H^C}}{M_{\rm cut}}\right) \sim s_{\rm loop}\left\{Y_N^{\rm I}{}^\dagger Y_N^{\rm I} - \frac{1}{2}\,s\left[Y_N^{\rm I}{}^\dagger C_1^{N_{\rm I}} + C_1^{N_{\rm I}}{}^\dagger Y_N^{\rm I}\right]\cdots\right\}, \quad (10{\cdot}5)$$

the effect of the shift $M_{H^C} \to M_{H^C}(1+\mathcal{O}(s))$ is

$$\frac{1}{16\pi^2}\boldsymbol{Y_N^{\rm I}}^\dagger\boldsymbol{Y_N^{\rm I}} \ln\left(\frac{M_{H^C}(1+\mathcal{O}(s))}{M_{H^C}}\right) \sim s \times s_{\rm loop}\left\{Y_N^{\rm I}{}^\dagger Y_N^{\rm I} + \cdots\right\}. \quad (10{\cdot}6)$$

This term is of the same order of the term with a square bracket in Eq. (10·5). Differently than that term, however, this is only a flavour-blind shift of the leading term $s_{\rm loop} Y_N^{\rm I\dagger} Y_N^{\rm I}$, which does not introduce an additional flavour structure. A simple treatment of the GUT threshold in which all superheavy masses are decoupled at the same scale also automatically disregards such effects.

In alternative to the effective-coupling picture, it is possible to use the procedure adopted by the authors of Ref. 19), in which each coupling of the various NROs is treated separately instead of being collected in a few effective couplings. We refer the reader to their work for details. As in our case, also their treatment is valid at most at $\mathcal{O}(s \times s_{\rm loop})$, and our analysis of boundary conditions for the NROs is valid also in their case.

10.2. *RGE treatment of effective couplings*

The evolution of effective couplings is analogous to that of couplings of renormalizable operators, even above $M_{\rm GUT}$ when the field 24_H is still active, provided the terms that break this picture are very small. As shown in the previous subsection,

these terms do exist, but can be neglected for a target accuracy of $\mathcal{O}(s \times s_{\rm loop})$ in the evaluation of sFVs.

The proof follows exactly the steps usually made when deriving the RGEs in renormalizable SUSY models. Since the superpotential is not renormalized, only the Kähler potential is subject to loop corrections, which determine the anomalous dimensions for the various fields. As a results of these corrections the Kähler potential looses its minimality. Its canonical (or nearly canonical) form can be recovered through field redefinitions, which affect also the superpotential. The parameters in the superpotential and Kähler potential, expressed in terms of the redefined fields, are modified with respect to the original ones. These modifications give rise to the RGEs for the model.

A detailed proof in this case of effective couplings is given in Appendix D. Here we concentrate only on some final steps of this proof, which allow a comparison of our analysis to others existing in the literature.

For simplicity, let us consider a generic superpotential term $\boldsymbol{Y}(24_H)\Phi_1\Phi_2\Phi_3$, with

$$\boldsymbol{Y}(24_H) = \sum_{n=0} C_n 24_H^n, \tag{10·7}$$

and suppressed SU(5) indices. The quantity $\mathbf{Y}(\langle 24_H \rangle)$ will become the effective coupling for the operator $\Phi_1\Phi_2\Phi_3$, once the field 24_H acquires its *vevs*.

Through the procedure outlined above, we obtain the following RGEs for the original couplings:

$$\dot{C}_n = \left(\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3 + n\boldsymbol{\gamma}_{24_H}\right) C_n, \tag{10·8}$$

in terms of effective couplings, on which the anomalous dimensions $\boldsymbol{\gamma}_i$ depend. We have neglected here the fact that C_n and the various $\boldsymbol{\gamma}_i$ in general do not commute. Since different C_n have different RGEs, with the ratio C_n/C_0 evolving as $n\boldsymbol{\gamma}_{24_H} C_n/C_0$, it seems that the anomalous dimension $\boldsymbol{\gamma}_{24_H}$ destroys the picture of effective couplings at the loop level. The largest effect of γ_{24} on the effective Yukawa couplings is of $\mathcal{O}(s \times s_{\rm loop})$. Through the Yukawa couplings, also the effective soft mass squared are affected, but the largest flavour-dependent effect is of $\mathcal{O}(s \times s_{\rm loop}^2)$, and can be neglected. There is, however, a larger effect induced by the anomalous dimension γ_{24} on the effective soft masses squared, of $\mathcal{O}(s \times s_{\rm loop})$. This is due to the wave-function renormalization on the n-th term in the expansion of these masses in powers of the field 24_H, analogous to the expansion for the Yukawa couplings. This effect is flavour independent, and therefore irrelevant for all practical purposes.

Similarly, for the coefficients of $\boldsymbol{A}_{F_{\sf X}}(24_H)$:

$$\boldsymbol{A}_{F_{\sf X}}(24_H) = \sum_{n=0} \widetilde{C}_n 24_H^n, \tag{10·9}$$

the part of the effective trilinear coupling corresponding to $\boldsymbol{Y}(24_H)$ (the only one considered in the literature), we obtain:

$$\dot{\widetilde{C}}_n = \left(\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3 + n\boldsymbol{\gamma}_{24_H}\right) \widetilde{C}_n + \left(\widetilde{\boldsymbol{\gamma}}_1 + \widetilde{\boldsymbol{\gamma}}_2 + \widetilde{\boldsymbol{\gamma}}_3 + n\widetilde{\boldsymbol{\gamma}}_{24_H}\right) C_n. \tag{10·10}$$

The picture of effective couplings seems now to be destroyed by both, $\boldsymbol{\gamma}_{24_H}$ and $\widetilde{\boldsymbol{\gamma}}_{24_H}$. Even assuming universal boundary conditions for all the $\widetilde{C}_n$ at the cutoff scale, $\widetilde{C}_n = A_n C_n$, with $A_n = A_0$, differences such as

$$\Delta A = A_1 - A_0, \tag{10·11}$$

are generated at the one-loop level, and a misalignment of $\mathcal{O}(s \times s_{\rm loop})$ between the effective Yukawa couplings and trilinear couplings is introduced by $\boldsymbol{\gamma}_{24_H}$ and $\widetilde{\boldsymbol{\gamma}}_{24_H}$, *i.e.* precisely of the order of the effects that we would like to retain in our calculation. This is in addition to the usual misalignment induced also in the MSSM at $\mathcal{O}(s_{\rm loop})$ by the corresponding RGEs, expressed by the fact that the ratio $(\widetilde{C}_0/C_0)$ evolves as $(\tilde{\gamma}_1 + \tilde{\gamma}_2 + \tilde{\gamma}_3)$. The former is smaller, by a factor of $\mathcal{O}(s)$, but it is not governed by the CKM elements as the latter one.

All these effects are fictitious. To begin with, the running of the *vev* v_{24} cancels exactly the terms $n\gamma_{24}$ in the RGEs for C_n, giving for the effective couplings:

$$\dot{\boldsymbol{Y}}|_n = (\dot{C_n v_{24}^n}) = (\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3)\, \boldsymbol{Y}|_n. \tag{10·12}$$

It cancels also the term $n\gamma_{24}$ in the RGE for $\widetilde{C}_n$, giving for $\boldsymbol{A}_{F_{\mathsf{X}}}|_n = (\widetilde{C}_n v_{24}^n)$:

$$\dot{\boldsymbol{A}}_{F_{\mathsf{X}}}|_n = (\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3)\, \boldsymbol{A}_{F_{\mathsf{X}}}|_n + \big(\widetilde{\boldsymbol{\gamma}}_1 + \widetilde{\boldsymbol{\gamma}}_2 + \widetilde{\boldsymbol{\gamma}}_3 + n\widetilde{\boldsymbol{\gamma}}_{24_H}\big)\, \boldsymbol{Y}|_n, \tag{10·13}$$

The term $n\widetilde{\boldsymbol{\gamma}}_{24_H}$ in this equation is also fictitious. It is cancelled by the running of the *vev* F_{24} in:

$$\boldsymbol{A}_{F_{24}}(24_H) = \sum_{n=0} n f_{24} C_n 24_H^n, \tag{10·14}$$

the other contribution to effective trilinear terms, neglected in the literature, whose n-th term evolves as:

$$\dot{\boldsymbol{A}}_{F_{24}}|_n = (n f_{24} \dot{C_n v_{24}^n}) = (\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 + \boldsymbol{\gamma}_3)\, \boldsymbol{A}_{F_{24}}|_n - n\widetilde{\boldsymbol{\gamma}}_{24_H} \boldsymbol{Y}|_n. \tag{10·15}$$

Thus, the correct equation for the complete effective trilinear coupling, $\boldsymbol{A}|_n = \boldsymbol{A}_{F_{\mathsf{X}}}|_n + \boldsymbol{A}_{F_{24}}|_n$, is recovered.

The picture of effective couplings is used at the quantum level also in Ref. 40). No evolution of the *vevs* of the field 24_H is considered there and the correct RGEs are obtained only as an approximation, simply by neglecting the terms $n\gamma_{24_H}$ and $n\tilde{\gamma}_{24_H}$ in Eqs. (10·8) and (10·10). The misalignment that these terms produce at $\mathcal{O}(s \times s_{\rm loop})$ between the effective trilinear couplings and effective Yukawa couplings is within the precision of our calculation and we cannot neglect it.

The authors of Ref. 19) use a different procedure for their analysis. We can, nevertheless try to discuss it, using our language of effective couplings. Their RGEs between $M_{\rm cut}$ and $M_{\rm GUT}$ for the parameters corresponding to our C_n and $\widetilde{C}_n$, coincide with those in our Eqs. (10·8) and (10·10). These authors therefore claim that a term ΔA, as in Eq. (10·11) is generated. As explained, this should be cancelled by the running of the *vevs* in the effective couplings, which they ignore, together with the running of the parameters in the Higgs sector. Thus, such a cancellation is not incorporated in their analysis. In particular, because the running of F_{24} is not

included, they do not realize that their vanishing value of F_{24} at $M_{\rm GUT}$, which we deduce from their matching condition at this scale, is incompatible with an equally vanishing value at $M_{\rm cut}$, which they seem to adopt. Thus, their effective trilinear couplings at $M_{\rm cut}$ contain already tree-level sFVs, which depend on the angle θ_D of the mismatch matrix that we call $\varDelta_D$ in Eqs. (4·18) and (4·19). (See Eq. (7·11).) We believe that the large dependence on this angle, which they find in the rate for $\mu \to e\gamma$ (see their Fig. 9), is due to sLFVs in their boundary values.

§11. Low energy sFVs from seesaw mechanism and NROs

The simplest scenario to deal with, and the one to which we restrict ourselves in this section, is the first discussed in Sec. 9. This emerges essentially by imposing that the universal couplings of the SUSY-breaking field X to the superpotential and Kähler potential are stable under the field redefinitions of Eq. (8·3), and that the NROs of dimension five can be eliminated from the Kähler potential. It requires the minimum number of soft parameters, guarantees universality of the effective soft masses squared and the alignment of trilinear effective couplings with effective Yukawa couplings.

The modifications induced in this scenario by NROs in the sFVs of the MSSU(5) model, through the effective Yukawa couplings, are twofold. On one side NROs contribute to sFVs through RGEs and alter the pattern of sFVs of the MSSU(5)-model at $\mathcal{O}(s \times s_{\rm loop})$.

The largest effect, however, is that of the mismatch matrices. The mismatch in diagonalization of the various effective Yukawa couplings generated by the same class of operators, for example $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^5_E$, $\boldsymbol{Y}^5_{LQ}$, and $\boldsymbol{Y}^5_{DU}$, in the case of $Op^{\bar{5}_M}$, affects also the various sectors of the sfermion mass matrices, thereby spoiling the correlations between sQFVs and sLFVs. It should be emphasized that these rotations can only modify already existing sFVs, but cannot generate them if they do not. (For example, the term $\widetilde{D}^c \widetilde{\boldsymbol{m}}^2_{D^c} \widetilde{D}^{c*}$ cannot be affected by mismatch matrices if $\widetilde{\boldsymbol{m}}^2_{D^c}$ is proportional to the unit matrix.) Moreover, since their entries, or at least their largest entries, can be of $\mathcal{O}(1)$, these mismatch matrices alter the patterns of the MSSU(5)-model sFVs, producing modifications that can be of the same order of the sFVs themselves.

Notice that there is only one mismatch matrix affecting in this indirect way the seesaw-induced sFVs in $\widetilde{\boldsymbol{m}}^2_{D^c}$, driven by $\boldsymbol{Y}^{\rm I}_{ND}$. This is the matrix $\varDelta_D$ defined in terms of $\varDelta V_{5\,D}$ and a diagonal matrix of phases in Eq. (4·10).

For the fields in the $\bar{5}_M$ sector, there also sFVs induced by $\boldsymbol{Y}^5_{DU}$ and $\boldsymbol{Y}^5_{LQ}$, which involve several mismatch matrices, and for which the level of arbitrariness is large. These sFVs are, however, considerably smaller than those induced by the seesaw couplings. Similarly, other mismatch matrices affect sFVs for fields in the 10_M representation, which are driven by $\boldsymbol{Y}^{10}_U$, $\boldsymbol{Y}^{10}_{QQ}$, $\boldsymbol{Y}^{10}_{UE}$. Thus, this sector is also plagued by a large degree of uncertainty/arbitrariness. This is true in particular for the right-right sector of the charged slepton mass matrix, which in the MSSU(5) model exhibits sFVs induced by the Yukawa couplings of the top quark.[65]

The importance of the mismatch matrices for breaking the correlation present

in the MSSU(5) model between the seesaw-induced sQFVs and sLFVs was strongly emphasized in Ref. 19). While the authors of this paper stressed this effect for the case of first-second generation transitions, it is easy to see that the correlations of the minimal model are also lost in the case of second-third and first-third generation transitions.[22] We show this explicitly, by integrating the relevant RGEs in an approximated way, that is, neglecting the scale dependence of their coefficients as well as the contributions from all other effective Yukawa couplings.

We start reviewing first the case of the MSSU(5) model. Since the evolution of the seesaw-induced off-diagonal elements of the two matrices $\widetilde{m}_L^{2\,*}$ and $\widetilde{m}_{D^c}^2$ is determined only by the interaction $-N^c Y_N^{\rm I} \bar{5}_M 5_H$ of Eq. (2·47), we find the following approximated expressions for their elements (i,j), with $i \neq j$:

$$
\begin{aligned}
\widetilde{m}^{2\,*}_{L\,(i,j)} &= (t_{\rm GUT}+t_{\rm ssw}) \times \mathcal{F}\big(Y_N^{{\rm I}T}, \widetilde{m}^2_{\bar{5}_M}, \widetilde{m}^{2\,*}_{N^c}, \widetilde{m}^2_{5_H}, A_N^{{\rm I}T}\big)_{(i,j)}\Big|_{Q=M_{\rm cut}}, \\
\widetilde{m}^{2}_{D^c\,(i,j)} &= \quad (t_{\rm GUT}) \quad \times \mathcal{F}\big(Y_N^{{\rm I}T}, \widetilde{m}^2_{\bar{5}_M}, \widetilde{m}^{2\,*}_{N^c}, \widetilde{m}^2_{5_H}, A_N^{{\rm I}T}\big)_{(i,j)}\Big|_{Q=M_{\rm cut}},
\end{aligned}
\tag{11·1}
$$

with the function $\mathcal{F}$ defined in Eq. (E·15). Here $t_{\rm GUT}$ and $t_{\rm ssw}$ are

$$
t_{\rm GUT} = \frac{1}{16\pi^2}\ln\left(\frac{M_{\rm GUT}}{M_{\rm cut}}\right), \qquad t_{\rm ssw} = \frac{1}{16\pi^2}\ln\left(\frac{M_{\rm ssw}}{M_{\rm GUT}}\right). \tag{11·2}
$$

If the assumption of universality of soft masses is made, the function $\mathcal{F}$ is

$$
\mathcal{F}\left(Y_N^{{\rm I}T}, \widetilde{m}^2_{\bar{5}_M}, \widetilde{m}^{2\,*}_{N^c}, \widetilde{m}^2_{5_H}, A_N^{{\rm I}T}\right)_{(i,j)}\Bigg|_{Q=M_{\rm cut}} = 2\left(3\widetilde{m}_0^2 + A_0^2\right)\left(Y_N^{{\rm I}T} Y_N^{{\rm I}*}\right)_{(i,j)}\Big|_{Q=M_{\rm cut}}, \tag{11·3}
$$

and the simple relation between sLFVs and some sQFVs is obtained:

$$
\widetilde{m}^2_{D^c\,(i,j)} = \left(1 + \frac{t_{\rm ssw}}{t_{\rm GUT}}\right)^{-1} \times \widetilde{m}^{2\,*}_{L\,(i,j)} \qquad (i \neq j). \tag{11·4}
$$

Thus, because the Higgs triplets decouple at $M_{\rm GUT}$, a couple of order of magnitude above $M_{\rm ssw}$, sLFVs are in absolute value larger than sQFVs.

An inspection of the decomposition of the term $\bar{5}_M Y_N^{\rm II} 15_H \bar{5}_M$ for the seesaw of type II (see Eq. (2·47)) shows that, in contrast, the full SU(5) interaction remains active down to $M_{\rm ssw}$. Thus, in the seesaw of type II, sLFVs and sQFVs are expected to be of the same order up to sub-leading SU(5)-breaking effects in the RG flows below $M_{\rm GUT}$. As for the type III, because two of the interactions inducing sLFVs and one of those inducing sQFVs survive between $M_{\rm GUT}$ and $M_{\rm ssw}$ (see the third decomposition in Eq. (2·47)) the relation between sLFVs and sQFVs depends on group-theoretical factors.

An explicit calculation is particularly simple within the same approximation used for the seesaw of type I. Indeed, relations similar to that shown in Eq. (11·4) are obtained, where $t_{\rm ssw}$ has to be replaced by the product of $t_{\rm ssw}$ and the functions $r_{\rm II}(t_{\rm ssw}, t_{\rm GUT})$ and $r_{\rm III}(t_{\rm ssw}, t_{\rm GUT})$, respectively. Including now the seesaw of type I, we define a function $r_{\rm ssw}(t_{\rm ssw}, t_{\rm GUT})$, which in the three cases, has the values:

$$
r_{\rm I} = 1, \qquad r_{\rm II} = 0, \qquad r_{\rm III} = -\left(24 + 10\frac{t_{\rm ssw}}{t_{\rm GUT}}\right)^{-1}. \tag{11·5}
$$

	type I	type II	type III (and I)
mediator	N^c	15_H	24_M
interaction	$N^c\bar{5}_M 5_H$	$\bar{5}_M 15_H \bar{5}_M$	$5_H 24_M \bar{5}_M$
only sLFVs	$N^c L H_u$ -	LTL -	$H_u W_M L$, $H_u B_M L$ $H_U^C X_M L$
sLFVs & sQFVs	-	$D^c L Q_{15}$	-
only sQFVs	- $N^c D^c H_U^C$	$D^c S D^c$ -	$H_u \bar{X}_M D^c$ $H_U^C G_M D^c$, $H_U^C B_M D^c$
|sLFVs/sQFVs|	> 1	~ 1	~ 1

Table I. *The SU(5) Yukawa interactions for the multiplets in which the seesaw mediators are embedded, together with their SM decompositions, listed in different lines depending on the type of flavour violation they originate. Signs and numerical coefficients reported in the text are omitted here. The expected size of sLFVs versus sQFVs, as defined in the text, are also given for the three types of the seesaw mechanism.*

The relation in Eq. (11·4) becomes then:

$$\widetilde{m}^2_{D^c\,(i,j)} = R_{\rm ssw} \times \widetilde{m}^{2\,*}_{L\,(i,j)} \qquad (i \neq j), \tag{11·6}$$

with

$$R_{\rm ssw} = \left[1 + r_{\rm ssw}(t_{\rm ssw}, t_{\rm GUT})\frac{t_{\rm ssw}}{t_{\rm GUT}}\right]^{-1}. \tag{11·7}$$

Since the ratio $t_{\rm ssw}/t_{\rm GUT}$ is assumed here to be $\lesssim 1$, $r_{\rm III}$ is at the percent level, and sLFVs and sQFVs are also of the same order, as in the seesaw of type II. We summarize the situation in table I, where we list the SU(5) Yukawa interactions for the multiplets in which the seesaw mediators are embedded, their SM decompositions (omitting signs and numerical coefficients) and the size of sLFVs versus sQFVs, defined as the ratio $|\widetilde{m}^2_{L\,(i,j)}/\widetilde{m}^2_{D^c\,(i,j)}|$ with $i \neq j$ for the three types of the seesaw mechanism. We assume in this table an exact integration of the relevant RGEs, which explain the value ~ 1 for this ratio in the case of the seesaw of type II. (The value $r_{\rm II} = 0$ was obtained in a particular approximation.)

In the nrMSSU(5) models, the parameters $\widetilde{m}^2_{D^c}$ and $\widetilde{m}^2_L$ are replaced by $\widetilde{\boldsymbol{m}}^2_{D^c}$ and $\widetilde{\boldsymbol{m}}^2_L$. For the seesaw of type I, a one-step integration of the corresponding RGEs gives for the off-diagonal elements of these matrices:

$$\begin{aligned}
\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(i,j)} &= (t_{\rm GUT}+t_{\rm ssw}) \times \mathcal{F}\big(\boldsymbol{Y}^{{\rm I}\,T}_N, \widetilde{\boldsymbol{m}}^{2\,*}_L, \widetilde{m}^{2\,*}_{N^c}, \widetilde{m}^2_{H_u}, \boldsymbol{A}^{{\rm I}\,T}_N\big)_{(i,j)}\Big|_{Q=M_{\rm cut}}, \\
\widetilde{\boldsymbol{m}}^2_{D^c\,(i,j)} &= (t_{\rm GUT}) \times \mathcal{F}\big(\boldsymbol{Y}^{{\rm I}\,T}_{ND}, \widetilde{\boldsymbol{m}}^2_{D^c}, \widetilde{m}^{2\,*}_{N^c}, \widetilde{m}^2_{H_U^C}, \boldsymbol{A}^{{\rm I}\,T}_{ND}\big)_{(i,j)}\Big|_{Q=M_{\rm cut}}.
\end{aligned} \tag{11·8}$$

Thus, the elements $\widetilde{\boldsymbol{m}}^2_{L\,(i,j)}$ are, to a good approximation,

$$\begin{aligned}
\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(i,j)} &= (t_{\rm GUT}{+}t_{\rm ssw}) \;\times\; 2\left(3\widetilde{m}_0^2+A_0^2\right)(\boldsymbol{Y}^{{\rm I}T}_N\boldsymbol{Y}^{{\rm I}*}_N)_{(i,j)}\big|_{Q=M_{\rm cut}}\,,\\
\widetilde{\boldsymbol{m}}^2_{D^c\,(i,j)} &= \quad (t_{\rm GUT}) \quad\;\times\; 2\left(3\widetilde{m}_0^2+A_0^2\right)(\boldsymbol{Y}^{{\rm I}T}_{ND}\boldsymbol{Y}^{{\rm I}*}_{ND})_{(i,j)}\big|_{Q=M_{\rm cut}}\,.
\end{aligned} \tag{11·9}$$

Since we have neglected NROs in the seesaw sectors, in SU(5)-symmetric bases, it is $\boldsymbol{Y}^{\rm I}_N=\boldsymbol{Y}^{\rm I}_{ND}=Y^{\rm I}_N$. After the SU(5)-breaking rotations $D^c\to P_{\rm I}^\dagger\varDelta_D D^c$ and $L\to e^{-i\phi_{\rm I}}P_{\rm I}^\dagger L$, we obtain the following expression for the elements $\widetilde{\boldsymbol{m}}^2_{D^c\,(i,j)}$:

$$\widetilde{\boldsymbol{m}}^2_{D^c\,(i,j)} = R_{\rm ssw}\times\left(\varDelta_D^T\;\widetilde{\boldsymbol{m}}^{2\,*}_L\;\varDelta_D^*\right)_{(i,j)}. \tag{11·10}$$

This is valid for all seesaw types. In particular, for the flavour transitions $(i,3)$, with $i=1,2$, we have:

$$\begin{aligned}
\widetilde{\boldsymbol{m}}^2_{D^c\,(i,3)} = R_{\rm ssw}\times\Big\{ &\varDelta_{D\,(1,i)}\,\varDelta^*_{D\,(2,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,2)}+\varDelta_{D\,(2,i)}\,\varDelta^*_{D\,(1,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,1)}\\
&+\varDelta_{D\,(1,i)}\,\varDelta^*_{D\,(3,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,3)}+\varDelta_{D\,(3,i)}\,\varDelta^*_{D\,(1,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(3,1)}\\
&+\varDelta_{D\,(2,i)}\,\varDelta^*_{D\,(3,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,3)}+\varDelta_{D\,(3,i)}\,\varDelta^*_{D\,(2,3)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(3,2)}\\
&+\varDelta_{D\,(i,i)}\;\varDelta^*_{D\,(i,3)}\Big[\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(i,i)}-\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(j,j)}\Big]\\
&+\varDelta_{D\,(3,i)}\,\varDelta^*_{D\,(3,3)}\Big[\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(3,3)}-\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(j,j)}\Big]\Big\},
\end{aligned} \tag{11·11}$$

with $j=(1,2)$ and $j\neq i$, where the unitarity of the matrix $(\varDelta_D)$ was used. If no hierarchies exist among the elements of the unitary matrix $\varDelta_D$, all terms in this equation contribute in the same way to the off-diagonal elements, $\widetilde{\boldsymbol{m}}^2_{D^c\,(i,3)}$. An equally complicated expression is obtained for $\widetilde{\boldsymbol{m}}^2_{D^c\,(1,2)}$ in terms of all elements in the matrix $\widetilde{\boldsymbol{m}}^2_L$ and of the mismatch matrix $\varDelta_D$, which we do not report here. We list instead the expression that both $\widetilde{\boldsymbol{m}}^2_{D^c\,(1,2)}$ and $\widetilde{\boldsymbol{m}}^2_{D^c\,(i,3)}$ get when the approximation of Sec. 4.3 is used:

$$\begin{aligned}
\widetilde{\boldsymbol{m}}^2_{D^c\,(1,2)} = R_{\rm ssw}\times\Big\{ &\varDelta_{D\,(1,1)}\,\varDelta^*_{D\,(2,2)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,2)}+\varDelta_{D\,(2,1)}\,\varDelta^*_{D\,(1,2)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,1)}\\
&+\varDelta_{D\,(1,1)}\,\varDelta^*_{D\,(1,2)}\Big[\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,1)}-\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(3,3)}\Big]\\
&+\varDelta_{D\,(2,1)}\,\varDelta^*_{D\,(2,2)}\Big[\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,2)}-\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(3,3)}\Big]\Big\},
\end{aligned} \tag{11·12}$$

$$\widetilde{\boldsymbol{m}}^2_{D^c\,(i,3)} = R_{\rm ssw}\times\Big\{\varDelta_{D\,(1,i)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,3)}+\varDelta_{D\,(2,i)}\,\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,3)}\Big\}\,\varDelta^*_{D\,(3,3)}. \tag{11·13}$$

Notice that here and in Eq. (11·11) the contributions coming from the diagonal elements of $\widetilde{\boldsymbol{m}}^2_L$ tend to cancel each others under the assumption of universal boundary conditions for the soft terms.

In the above relations, the prediction of the MSSU(5) model is recovered for $(\varDelta_D)_{(1,2)}\sim 0$, but it is strongly violated if $(\varDelta_D)_{(1,2)}\sim(\varDelta_D)_{(1,1)}$. Even in such a case, there is a relation between elements of $\widetilde{\boldsymbol{m}}^2_{D^c}$ and $\widetilde{\boldsymbol{m}}^2_L$ that remains unchanged

by the rotation matrix Δ_D:

$$\left[\left|\widetilde{\boldsymbol{m}}^2_{D^c\,(1,3)}\right|^2 + \left|\widetilde{\boldsymbol{m}}^2_{D^c\,(2,3)}\right|^2\right]^{1/2} = R_{\rm ssw} \times \left[\left|\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(1,3)}\right|^2 + \left|\widetilde{\boldsymbol{m}}^{2\,*}_{L\,(2,3)}\right|^2\right]^{1/2}, \qquad (11\cdot14)$$

reported also in Ref. 22).

§12. Summary

We have analyzed the issue of flavour and CP violation induced by seesaw Yukawa couplings in the low-energy soft parameters of the MSSU(5) model with NROs, or nrMSSU(5) models, with the seesaw mechanism. Although our way of dealing with these NROs at the quantum level is closer to the procedure proposed in Ref. 40), we have largely built on the work of Ref. 19), the first to deal in this context with NROs to correct the erroneous predictions of the MSSU(5) model for fermion masses. The addition of NROs is a minimal ultraviolet completion sufficient to rescue the MSSU(5) model also from the other fatal problem by which it is plagued, *i.e.* that of predicting a rate for the decay of the proton that is too fast.

We have gone beyond the analysis of Ref. 19) in several directions. Differently than in Ref. 19), we have not tried to restrict the number and type of NROs introduced, complying with the idea that, barring accidentally vanishing couplings, it is difficult to confine NROs to a certain sector of the model, or impose that they are of a certain type. Their physical relevance, however, decreases when their dimensionality increases and, depending on the accuracy required for the problem under consideration, NROs of certain dimensions can be completely negligible. For our calculation of sFVs at a precision of $\mathcal{O}(s \times s_{\rm loop})$, it is sufficient to include NROs of dimension five at the quantum level and the tree-level boundary values of NROs of dimension six. In spite of this, we have kept our discussion as general as possible.

Since their impact is of little consequence for sFVs, we have also disregarded the effects of NROs in the Higgs sector. This approximation has allowed us to neglect possible modifications of the vacuum structure, which is, therefore, that of the MSSU(5) model, although a generalization to the case with nonvanishing NROs in this sector is conceptually straightforward. We have studied this vacuum in detail, emphasizing the scaling behaviour of the scalar and auxiliary *vevs* v_{24} and F_{24} of the field 24_H. We have also shown what an important role these two *vevs* play for the determination of the boundary conditions of various soft parameters and for their evolution.

We have introduced the concept of effective couplings, which get contributions from the couplings of renormalizable operators and those of nonrenormalizable ones when the field 24_H acquires *vevs.* This concept can be applied to the case of other GUT groups (it was indeed introduced in Ref. 40), for a SUSY SO(10) model) and can also be used when the superheavy Higgs fields acquiring *vevs* are not in the adjoint representation.

By making use of these couplings, we have analyzed in full generality the amount of arbitrariness introduced by NROs in the Yukawa sector, which in general spoils the correlation between the seesaw-induced sQFVs and sLFVs typical of the MSSU(5)

model. This arbitrariness can be parametrized by several diagonal matrices of effective Yukawa couplings and several matrices of diagonalization mismatch, each of them expressed in terms of mixing angles and phases. The parameter space opened up by the introduction of NROs in this sector is, indeed, quite large. It is precisely this arbitrariness that allows to slow down the decay rate of the proton.

We have pointed out that the suppression of this rate through NROs in the Yukawa sector may have an important feedback for the evaluation of sFVs. We have shown that the correlation between the seesaw-induced sQFVs and sLFVs typical of the MSSU(5) model, remains practically unchanged in a particular point of the above parameter space in which the decay rate of the proton is suppressed. We have also pointed out that different NROs, in addition to those in the Yukawa sector, could yield the necessary suppression of the proton-decay rate and argued, as a consequence, that the suppression of this rate by NROs does not necessarily guarantee that the pattern of sFVs in nrMSSU(5) models remains that of the MSSU(5) one.

The concept of effective couplings has allowed us to formulate in a general way the problem of possible sFVs at the tree level, *i.e.* in the boundary conditions of soft parameters, which NROs in general introduce. It has provided a tool to address the problem of finding conditions to be imposed on the couplings of the SUSY-breaking mediator X to the superpotential and Kähler potential to avoid such tree-level sFVs. (See Secs. 7.1 and 7.2.) This is because the usual requirement of universality of these couplings, that is, of blindness to flavour and field type is not sufficient in the context of models with NROs. We have called this blindness weak universality.

We have shown that the needed special conditions can actually be obtained in a general way in models with NROs, by imposing that this universality of couplings of X to the superpotential and Kähler potential are stable under the field redefinitions of Eq. (8·3), and that the dimension-five operators in the Kähler potential can be removed. This has lead us to identify a special type of universal couplings of X to the superpotential and Kähler potential, or strongly-universal couplings, which amounts to having, in the same basis, the factorizable form of Eqs. (8·8) and (8·18) for these two potentials. Clearly, in models with vanishing NROs, such as the MSSM, the two notions of universality coincide.

When this type of strong universality is advocated, the soft SUSY-breaking terms can be described by the usual four parameters, $\widetilde{m}_0^2$, A_0, B_0 and $M_{1/2}$, and the only arbitrariness induced by NROs in the predictions of the seesaw-induced sFVs is that of the Yukawa sector. If we impose $(B_0 - A_0)^2 = \widetilde{m}_0^2$ the situation is like that emerging by taking the flat limit of models embedded in minimal supergravity. In both cases, an ambiguity appears at the GUT scale in the determination of the MSSM B parameter, depending on how the tuning for this and the MSSM μ parameters are made.

We have found that it is possible to extend the picture of effective couplings to the quantum level, obviously not in general, but within certain limits. These limits restrict the accuracy that can be obtained for the evaluation of sFVs in nrMSSU(5) models treated in this way. The maximal accuracy that we can achieve is of $\mathcal{O}(s \times s_{\rm loop})$. Nevertheless, this is more than adequate for our study of sFVs. We have shown that within this accuracy, the nrMSSU(5) models can be treated like

renormalizable ones, as the effective couplings do obey RGEs typical of renormalizable models. Clearly this is not true for the original couplings of the various NROs. The evolution of the *vevs* of the field 24_H, however, corrects for the difference in the running of these original couplings, which enter in the definition of the effective couplings, and the running of the effective couplings themselves. As shown in Appendix C, this fact is completely general, and it applies to: *i)* other GUT groups, *ii)* the case in which the fields that acquire *vevs* are in representations different than the adjoint, *iii)* the case in which NROs are included in the Higgs sector.

Within the accuracy of our analysis, the one-loop results obtained with this method are not different from those obtained with the more conventional method of Ref. 19), when the running of the two *vevs* v_{24} and F_{24} is kept into account. The effect of this running is missed in the existing literature.[19),22)] (See for example the discussion at the end of Sec. 10.2.)

We have insisted on this effective picture as it gives, in our opinion, a very clear physical interpretation of the parameters of these models and include the running of the *vevs* in a natural way.

We have given complete lists of RGEs for the nrMSSU(5) models, together with those for the MSSU(5) model, and for the MSSM, for all the three types of the seesaw mechanism. (This last set of evolution equations are needed at scales below $M_{\rm GUT}$.) Some of these RGEs are presented here for the first time. The others, provide a check of RGEs existing in the literature, which we correct, when possible.

Finally, by making use of approximated solutions of the above RGEs, we have sketched how the predictions for the seesaw-induced sFVs can be modified by the presence of NROs in the Yukawa sector, in the scenario of strongly-universal couplings of X to the Kähler potential and superpotential. Modifications are induced not only in first-second generation sFVs, but also in those involving the third generation.

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Appendix A

—— *Embeddings in GUT representations: normalizations and group factors* ——

Lepton and Higgs doublets of SU(2) are contained in 5plets of SU(5); antidoublets in $\bar{5}$plets, with the antidoublet of, say, L given by ϵL. We remind that ϵ is the 2×2 matrix proportional to the Pauli matrix σ_2 ($\epsilon = i\sigma_2$), with elements $\epsilon_{11} = \epsilon_{22} = 0$, $\epsilon_{12} = -\epsilon_{21} = 1$. Thus, the SM decompositions of $\bar{5}_M$, 5_H and $\bar{5}_H$ are:

$$(\bar{5}_M)_A = \begin{pmatrix} D^c_a \\ (\epsilon L)_\alpha \end{pmatrix}, \qquad (5_H)_A = \begin{pmatrix} (H^C_U)_a \\ (H_u)_\alpha \end{pmatrix}, \qquad (\bar{5}_H)_A = \begin{pmatrix} (H^C_D)_a \\ (\epsilon H_d)_\alpha \end{pmatrix}, \quad \text{(A·1)}$$

where A, like all SU(5) indices denoted with upper case latin letters, is decomposed into SU(3) and SU(2) indices, denoted respectively by lower case latin letters and lower case greek ones. The 10plet of SU(5) is usually expressed as a 5×5 antisymmetric matrix:

$$(10_M)_{AB} = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_{abc} U^c_c & -Q^T_{a\beta} \\ Q_{\alpha b} & -\epsilon_{\alpha\beta} E^c \end{pmatrix}, \quad \text{(A·2)}$$

with ϵ_{abc} antisymmetric in the exchange of two contiguous indices, and $\epsilon_{123} = 1$. The multiplication rule $10^*_M 10_M = (10^*_M)_{AB}(10_M)_{AB} = \mathrm{Tr}(10^\dagger_M 10_M)$, together with the coefficient $1/\sqrt{2}$, guarantees that the kinetic terms for 10_M are correctly normalized.

The SU(2) doublets Q, L, H_u and H_d in the previous two equations are, as usual,

$$Q = \begin{pmatrix} U \\ D \end{pmatrix}, \qquad L = \begin{pmatrix} N \\ E \end{pmatrix}, \qquad H_d = \begin{pmatrix} H^0_d \\ H^-_d \end{pmatrix}, \qquad H_u = \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}. \quad \text{(A·3)}$$

The SU(5) multiplication of a 5 and a $\bar{5}$ multiplet, such as in $W_{\rm RHN}$ in Eq. (2·46), is understood to be trivially $\bar{5}_M 5_M \equiv (\bar{5}_M)_A (5_M)_A$. The first two terms of $W_{\rm M}^{\rm MSSU(5)}$ in Eq. (2·2) are

$$\begin{aligned} \bar{5}_M 10_M \bar{5}_H &\equiv (\bar{5}_M)_A (10_M)_{AB} (\bar{5}_H)_B, \\ 10_M 10_M 5_H &\equiv \epsilon_{ABCDE} (10_M)_{AB} (10_M)_{CD} (5_H)_E, \end{aligned} \quad \text{(A·4)}$$

with a fully antisymmetric ϵ_{ABCDE} and $\epsilon_{12345} = 1$. These multiplication rules and the embedding of doublets and antidoublets in 5 and $\bar{5}$ multiplets specified above, are consistent with the usual multiplication rule for SU(2) doublets, $LH_d \equiv L_\alpha \epsilon_{\alpha\beta} (H_d)_\beta$, and the SU(3) multiplication, $D^c U^c H^C_D \equiv \epsilon_{abc} D^c_a U^c_b (H^C_D)_c$.

Given all the above definitions, it is easy to see that the SM decomposition of the two terms in $W_{\rm M}^{\rm MSSU(5)}$ is

$$\begin{aligned} \sqrt{2}\, \bar{5}_M Y^5 10_M \bar{5}_H &= -D^c Y^5 Q H_d - E^c Y^{5\,T} L H_d - L Y^5 Q H^C_D - D^c Y^5 U^c H^C_D, \\ -\frac{1}{4} 10_M Y^{10} 10_M 5_H &= \ U^c Y^{10} Q H_u + \frac{1}{2} Q Y^{10} Q H^C_U + U^c Y^{10} E^c H^C_U; \end{aligned} \quad \text{(A·5)}$$

that for the Yukawa interaction in $W_{\rm RHN}$ can be read off from Eq. (2·47).

The Higgs field 24_H is accommodated into the traceless 5×5 matrix:

$$(24_H)_{AB} = \begin{pmatrix} (G_H)_{ab} & (X_H)_{\alpha\beta} \\ (\epsilon\bar{X}_H)_{\alpha b} & (\epsilon W_H)_{\alpha\beta} \end{pmatrix} + B_H\sqrt{\frac{6}{5}}\begin{pmatrix} \frac{1}{3}\delta_{ab} & 0 \\ 0 & -\frac{1}{2}\delta_{\alpha\beta} \end{pmatrix}, \tag{A·6}$$

where the subscript index H distinguishes its components from those of the gauge field in the adjoint representation 24, G, X, $\bar{X}$, W, and B. We define $(24_H)_{AB} \equiv \sqrt{2}\,24^i_H T^i_{AB}$ ($i = 1,...,24$), and similarly $(G_H)_{ab} \equiv \sqrt{2}\,G^i_H T^i_{ab}$ ($i = 1,...,8$) and $(W_H)_{\alpha\beta} \equiv \sqrt{2}\,W^i_H T^{i+20}_{\alpha\beta}$ ($i = 1,2,3$). The symbols T^i denote, as usual, SU(5) generators, with $\mathrm{Tr}T^iT^j = (1/2)\delta_{ij}$. The field X_H is an SU(2) doublet and an SU(3) antitriplet and has hypercharge 5/6; $\bar{X}_H$, with hypercharge $-5/6$, is an antidoublet of SU(2) and a triplet of SU(3). The field B_H, or 24^{24}_H, is a SM singlet, and $\sqrt{2}T^{24}_{AB}$ is the quantity $\sqrt{6/5}\,\mathrm{diag}(1/3,1/3,1/3,-1/2,-1/2)$ that multiplies it in Eq. (A·6). It is the field B_H that acquires the SU(5)-breaking *vev* $\langle B_H\rangle$:

$$B_H \to \langle B_H\rangle + B'_H. \tag{A·7}$$

In the limit of exact SUSY this *vev* coincides with v_{24} of Eq. (2·13).

With these ingredients it is easy to obtain the SM decomposition of the terms in $W_{\mathrm{H}}^{\mathrm{MSSU(5)}}$ (Eq. (2·3)). The first two terms give rise to:

$$\mu_3 H^C_U H^C_D + \mu_2 H_u H_d, \tag{A·8}$$

with μ_2 and μ_3 defined in Eqs. (2·14) and (2·15), and to:

$$\lambda_5\left\{H^C_U\left[G_H + \sqrt{\frac{6}{5}}\frac{1}{3}B'_H\right]H^C_D + \left[H^C_U X_H H_d + H_u\bar{X}_H H^C_D\right]\right.$$
$$\left. + H_u\left[W_H - \sqrt{\frac{6}{5}}\frac{1}{2}B'_H\right]H_d\right\}. \tag{A·9}$$

Finally, for the products of 24_H, we remind that:

$$\begin{aligned}(24_H)^2 &= \mathrm{Tr}(24_H\,24_H),\\ (24_H)^3 &= (1/2)\mathrm{Tr}(\{24_H,24_H\}24_H) = \mathrm{Tr}(24_H\,24_H\,24_H).\end{aligned} \tag{A·10}$$

The last relation is not general, but applies to the specific case considered here. Thus, for the last two terms in $W_{\mathrm{H}}^{\mathrm{MSSU(5)}}$, we obtain the decompositions:

$$\begin{aligned}{24_H}^2 =&\quad {B_H}^2 + \sum_i (G^i_H)^2 + \sum_i (W^i_H)^2 + \left(X_H\bar{X}_H - \bar{X}_H X_H\right),\\ {24_H}^3 =& -\frac{1}{\sqrt{30}}{B_H}^3 + 3\sqrt{\frac{6}{5}}B_H\\ &\times\left[\frac{1}{3}\sum_i (G^i_H)^2 - \frac{1}{2}\sum_i (W^i_H)^2 + \frac{1}{3}X_H\bar{X}_H + \frac{1}{2}\bar{X}_H X_H\right] + \cdots .\end{aligned} \tag{A·11}$$

Once the field B_H acquires its *vev*, the superpotential mass terms for the fields B'_H, G^i_H, and W^i_H become, in the supersymmetric limit:

$$-\frac{1}{2}M_{24}\left(B'_H\right)^2+\frac{1}{2}\left(5M_{24}\right)\sum_i(G^i_H)^2-\frac{1}{2}\left(5M_{24}\right)\sum_i(W^i_H)^2, \qquad \text{(A·12)}$$

whereas the Goldstone bosons X_H and $\bar{X}_H$ remain massless.

The matter fields 24_M are as 24_H, with the subscript index $_H$ replaced by $_M$ everywhere. Three such fields are introduced for the implementation of the seesaw of type III, labelled by a flavour index, which is suppressed in this appendix. The decomposition of the mass term $24_M M_{24_M} 24_M$, appearing in $W_{24\mathrm{M}}$ of Eq. (2·46), follows that of 24^2_H. The product $24_M 24_M 24_H$ in the last term of $W_{24\mathrm{M}}$, in contrast, is not as simple as the product $(24_H)^3$ discussed above. This is because there are, in reality, two ways to obtain a singlet out of the product of three adjoint representations of SU(5). Indeed, the product of two 24's is

$$24\times 24=1+24_S+24_A+\cdots, \qquad \text{(A·13)}$$

where 24_S and 24_A correspond, respectively, to a symmetric and an antisymmetric product. In the case of $(24_H)^3$, the antisymmetric part vanishes. In contrast, since more than one field 24_M exist, the antisymmetric product of two different 24_M does not vanish, and $24_M 24_M 24_H$ gets contribution form both, the symmetric and the antisymmetric product:

$$(24^2_M 24_H)=(1/2)\mathrm{Tr}(\{24_M,24_M\}\,24_H)+(1/2)\mathrm{Tr}([24_M,24_M]\,24_H). \qquad \text{(A·14)}$$

Thus, two distinct interaction terms with different Yukawa couplings $Y^S_{24_M}$ and $Y^A_{24_M}$ exist. The decomposition of the term $5_H 24_M Y^{\mathrm{III}}_N \bar{5}_M$ in $W_{24\mathrm{M}}$ is given in Eq. (2·47), where the terms $H_u W_M H_d$ and $H_u B_M H_d$ are understood as $H_u^T \epsilon W_M \epsilon H_d$ and $H_u^T B_M \epsilon H_d$, respectively.

The 15_H and $\bar{15}_H$ representations are expressed as 5×5 symmetric matrices:

$$(15_H)_{AB}=\begin{pmatrix} S_{ab} & \frac{1}{\sqrt{2}}(Q^T_{15})_{a\beta} \\ \frac{1}{\sqrt{2}}(Q_{15})_{\alpha b} & T_{\alpha\beta}\end{pmatrix},$$

$$(\bar{15}_H)_{AB}=\begin{pmatrix} \bar{S}_{ab} & \frac{1}{\sqrt{2}}(\bar{Q}^T_{15}\epsilon^T)_{a\beta} \\ \frac{1}{\sqrt{2}}(\epsilon\bar{Q}_{15})_{\alpha b} & (\epsilon\bar{T}\epsilon^T)_{\alpha\beta}\end{pmatrix}. \qquad \text{(A·15)}$$

In these matrices, S and $\bar{S}$ are singlets of SU(2) and a 6plet and $\bar{6}$plet of SU(3), respectively; Q_{15} has the same SU(3) and SU(2) quantum numbers of Q in Eq. (A·3), $\bar{Q}_{15}$ the opposite ones:

$$Q_{15}=\begin{pmatrix} U_{15} \\ D_{15}\end{pmatrix}, \qquad \bar{Q}_{15}=\begin{pmatrix} -\bar{D}_{15} \\ \bar{U}_{15}\end{pmatrix}, \qquad \text{(A·16)}$$

T and $\bar{T}$ are singlets of SU(3) and a triplet and an antitriplet of SU(2), respectively. Explicitly:

$$T = \begin{pmatrix} T^{++} & \frac{1}{\sqrt{2}}T^{+} \\ \frac{1}{\sqrt{2}}T^{+} & T^{0} \end{pmatrix}, \qquad \bar{T} = \begin{pmatrix} \bar{T}^{0} & \frac{1}{\sqrt{2}}\bar{T}^{-} \\ \frac{1}{\sqrt{2}}\bar{T}^{-} & \bar{T}^{--} \end{pmatrix}. \tag{A·17}$$

The factor $1/\sqrt{2}$ in the off-diagonal elements of 15_H and $\bar{15}_H$ is introduced in order to obtain the correct normalization of the kinetic terms for 15_H and $\bar{15}_H$. We remind that

$$\begin{aligned} 15_H^* 15_H &= (15_H^*)_{AB}(15_H)_{AB}, \\ \bar{15}_H^* \bar{15}_H &= (\bar{15}_H^*)_{AB}(\bar{15}_H)_{BA}, \\ 15_H \bar{15}_H &= (15_H)_{AB}(\bar{15}_H)_{BA}, \\ \bar{5}_M 15_H \bar{5}_M &= (\bar{5}_M)_A(15_H)_{AB}(\bar{5}_M)_B, \\ 15_H 24_H \bar{15}_H &= (15_H)_{AB}(24_H)_{BC}(\bar{15}_H)_{CA}, \end{aligned} \tag{A·18}$$

with similar rules hold for $5_H\bar{15}_H 5_H$ and $\bar{5}_H 15_H \bar{5}_H$.

Thus, the bilinear combination of the fields 15_H and $\bar{15}_H$ in the mass term $M_{15}15_H\bar{15}_H$ in $W_{\rm 15H}$ in Eq. (2·46) is

$$S\bar{S} + Q_{15}\bar{Q}_{15} + T\bar{T}, \tag{A·19}$$

where $T\bar{T} = T\epsilon\bar{T}\epsilon^T$. The SM decomposition of the first interaction in $W_{\rm 15H}$ can be read off from Eq. (2·46). The remaining terms proportional to λ_D and λ_U in $W_{\rm 15H}$ are

$$\begin{aligned} \frac{1}{\sqrt{2}}\lambda_D \bar{5}_H 15_H \bar{5}_H &= \frac{1}{\sqrt{2}}\lambda_D \left\{ H_D^C S H_D^C + \sqrt{2} H_D^C Q_{15} H_d + H_d T H_d \right\}, \\ \frac{1}{\sqrt{2}}\lambda_U 5_H \bar{15}_H 5_H &= \frac{1}{\sqrt{2}}\lambda_U \left\{ H_U^C \bar{S} H_U^C - \sqrt{2} H_D^C \bar{Q}_{15} H_u + H_u \bar{T} H_u \right\}, \end{aligned} \tag{A·20}$$

where the SU(2) multiplications of H_dTH_d (as well as of LTL in Eq. (2·47)) and of $H_u\bar{T}H_u$ are understood to be: $H_d^T\epsilon^T T\epsilon H_d$ and $H_u^T\epsilon\bar{T}\epsilon^T H_u$.

As for the proton-decay NROs in Eq. (3·19), the product of matter fields is understood as:

$$\begin{aligned} &[(10_M)_h(10_M)_i][(\bar{5}_M)_j(10_M)_k] = \\ &\qquad \epsilon_{ABCDX}[((10_M)_{AB})_h((10_M)_{CD})_i][((\bar{5}_M)_E)_j((10_M)_{EX})_k], \end{aligned} \tag{A·21}$$

with all SU(5) indices A, B, C, D, X summed over $\{1,2,3,4,5\}$. The fields 24_H in this NRO are understood to be inserted anywhere along the string of matter fields. That is, $Op^{\rm PD}$ share the same type of complication that Op^{10} has.

The NROs in Op^{10} are, indeed,

$$\begin{aligned} Op^{10} = -\frac{1}{4}\epsilon_{ABCDE}\,(10_M)_{A'B'}(10_M)_{C'D'}(5_H)_{E'} \sum_{\sum n_i = 0}^{k} \left(\frac{1}{M_{\rm cut}}\right)^{(\sum n_i)} \\ \times C^{10}_{n_1,n_2,n_3,n_4,n_5}\,(24_H)^{n_1}_{A'A}\,(24_H)^{n_2}_{B'B}\,(24_H)^{n_3}_{C'C}\,(24_H)^{n_4}_{D'D}\,(24_H)^{n_5}_{E'E}, \end{aligned} \tag{A·22}$$

where it is assumed:

$$(24_H)^n_{AB} = \delta_{AB} \qquad \text{for } n = 0. \tag{A·23}$$

Notice that when $k = 1$, *i.e.* in the case of NROs of dimension five, the five terms in the above sum are not all independent: they reduce to the two terms of $Op^{10}|_5$ in Eq. (3·3). The number of independent terms, however, increases rapidly with the number of dimensions. There are five, when $k = 2$:

$$\begin{aligned} Op^{10}|_6 = -\frac{1}{4M_{\rm cut}} \Big[& 10_M\, C^{10}_{2;a}\, (24_H)^2\, 10_M 5_H \;+\; 10_M\, C^{10}_{2;b}\, 10_M\, 5_H\, (24_H)^2 \\ & + 10_M\, C^{10}_{1,1;a}\, (24_H)\, 10_M\, (24_H)\, 5_H \\ & + 10_M\, C^{10}_{1,1;b}\, (24_H)\, 10_M\, 5_H\, (24_H) \\ & + 10_M\, C^{10}_{1,1;c}\, (24^T_H) 10_M\, (24_H)\, 5_H \Big], \end{aligned} \tag{A·24}$$

with coefficients that are linear combinations of the original $C^{10}_{n_1,n_2,n_3,n_4,n_5}$'s.

The contractions of SU(5) indices for the NROs in $Op^5|_5$ and $Op^{10}|_5$ is

$$\begin{aligned} \bar{5}_M 24^T_H 10_M \bar{5}_H &= (\bar{5}_M)_A (24_H)_{BA} (10_M)_{BC} (\bar{5}_H)_C, \\ \bar{5}_M 10_M 24_H \bar{5}_H &= (\bar{5}_M)_A (10_M)_{AB} (24_H)_{BC} (\bar{5}_H)_C, \\ 10_M 10_M 24_H 5_H &= \epsilon_{ABCDE} (10_M)_{AB} (10_M)_{CD'} (24_H)_{D'D} (5_H)_E, \\ 10_M 10_M 5_H 24_H &= \epsilon_{ABCDE} (10_M)_{AB} (10_M)_{CD} (5_H)_{E'} (24_H)_{E'E}. \end{aligned} \tag{A·25}$$

Those for the NROs in $Op^{10}|_6$ are at this point a simple exercise.

Appendix B

—— *Flavour rotations and identification of SM fields* ——

We remind that two different unitary matrices are needed to diagonalize a generic matrix Y_G, whereas only one is needed to diagonalize a symmetric one Y_S:

$$\widehat{Y}_G = U^T_G Y_G V_G, \qquad \widehat{Y}_S = W^T_S Y_S W_S, \tag{B·1}$$

and that any unitary matrix can be parametrized as follows:

$$U = e^{i\phi_U} P^{(1)}_U K_U P^{(2)}_U. \tag{B·2}$$

Here K_U and $P^{(i)}_U$ $(i = 1, 2)$, like all K- and P-matrices in this paper, are respectively a unitary matrix with three mixing angles and one phase (the CKM matrix is of this type) and diagonal phase matrices with two nonvanishing phases and with $\det P^{(i)}_U = 1$; ϕ_U is an overall phase.

Thus, Y_G and Y_S can be recast as:

$$\begin{aligned} Y_G &= e^{i\phi_G} \left(K_{U_G} P^{(2)}_{U_G} \right)^T \widehat{Y}_G\, P_G \left(K_{V_G} P^{(2)}_{V_G} \right), \\ Y_S &= e^{i\phi_S} \left(K_S P^{(2)}_S \right)^T \; \widehat{Y}_S\, P_S \left(K_S P^{(2)}_S \right). \end{aligned} \tag{B·3}$$

This is nothing but a reshuffling of the parameters of Y_G and Y_S. The eighteen independent parameters of Y_G are distributed among the three real and positive eigenvalues collected in $\widehat{Y}_G$, six mixing angles in K_{U_G} and K_{V_G}, and nine phases: two in each of the matrices $P^{(2)}_{U_G}$, $P^{(2)}_{V_G}$, P_G, one in each of the two matrices K_{U_G} and K_{V_G} and one in the overall factor $e^{i\phi_G}$. Similarly, the twelve parameters of Y_S are distributed among three real and positive eigenvalues, three mixing angles in K_S, and six phases, four in P_S and $P^{(2)}_S$, one in K_S, and one in the overall factor.

If the Yukawa matrices Y^5 and Y^{10} in $W_{\rm M}^{\rm MSSU(5)}$ in Eq. (2·2) are diagonalized as:

$$\widehat{Y}^5 = V_5^T Y^5 V_{10}, \qquad \widehat{Y}^{10} = W_{10}^T Y^{10} W_{10}, \tag{B·4}$$

the following flavour rotations of the complete $SU(5)$ multiplets $\bar{5}_M$, 10_M:

$$\bar{5}_M \to V_5 \bar{5}_M, \qquad 10_M \to V_{10} 10_M, \tag{B·5}$$

reduce the first interaction term of $W_{\rm M}^{\rm MSSU(5)}$ to be diagonal. By expressing the unitary matrix $W_{10}^\dagger V_{10}$ as

$$W_{10}^\dagger V_{10} \;=\; P^{(1)}_{10} K_{10} P^{(2)}_{10} e^{i\phi_{10}} \;\equiv\; U_{\rm CKM}, \tag{B·6}$$

where K_{10} will be identified with the high-scale CKM matrix, and by further re-defining $\bar{5}_M$ and 10_M as:

$$\bar{5}_M \to e^{i\phi_{10}} P^{(2)}_{10} \bar{5}_M, \qquad 10_M \to e^{-i\phi_{10}} P^{(2)\,\dagger}_{10} 10_M, \tag{B·7}$$

we obtain the form of $W_{\rm M}^{\rm MSSU(5)}$ given in Eq. (2·4).

The choice of the SU(5)-breaking rotation of Eq. (2·6) is specifically made to have the CKM matrix in the up-quark sector of the MSSM, as K_{10} is in the 10_M-sector of the MSSU(5) model, whereas the rotation:

$$\bar{5}_M = \{D^c, e^{-i\phi_l} P_l^\dagger L\}, \qquad 10_M = \{K_{10}^\dagger Q, K_{10}^\dagger P_{10}^\dagger U^c, e^{i\phi_l} P_l E^c\}, \tag{B·8}$$

has the effect of shifting the CKM matrix in the down squark sector of the MSSM:

$$W'^{\,\rm MSSM} = U^c\, \widehat{Y}_U\, QH_u - D^c \left(\widehat{Y}_D K_{\rm CKM}^\dagger\right) QH_d - E^c\, \widehat{Y}_D\, LH_d. \tag{B·9}$$

The flavour bases in this equation and in Eq. (2·8) are the most commonly used in the MSSM. The basis in which $W'^{\,\rm MSSM}$ has the form:

$$W'^{\,\rm MSSM} = U^c \left(K_{\rm CKM}^T \widehat{Y}_U K_{\rm CKM}\right) QH_u - D^c\, \widehat{Y}_D\, QH_d - E^c\, \widehat{Y}_D\, LH_d, \tag{B·10}$$

is used in Ref. 45). (See Sec. 5.) Except for the removal of the phases in P_{10}, this special form does not require SU(5)-breaking rotations to match the MSSU(5) model with the MSSM at $M_{\rm GUT}$.

Notice that we could have also chosen to parametrize Y^5 and Y^{10} in $W_M^{\rm MSSU(5)}$ in such a way to have the matrix K_{10} in the $\bar{5}_M$ sector:

$$W_{\rm M}^{\rm MSSU(5)} = \sqrt{2}\, \bar{5}_M \left(\widehat{Y}^5 K_{10}^\dagger P_{10}^{(1)}\right) 10_M \bar{5}_H - \frac{1}{4} 10_M\, \widehat{Y}^{10}\, 10_M 5_H. \tag{B·11}$$

For this, it is sufficient to rotate the field 10_M with the matrix W_{10} instead than with V_{10} as done above, and $\bar{5}_M$ as in Eq. (B·7). Starting from this superpotential, which basis for the up- and down-quark sector for the MSSM superpotential should be used is again a matter of choice.

Of the two complex matrices in $W_{\rm RHN}$ in Eq. (2·46), $Y_N^{\rm I}$ is a generic complex matrix, M_N, a symmetric one, and are diagonalized as follows:

$$\widehat{Y}_N^{\rm I} = U_N^{{\rm I}\,T} Y_N^{\rm I} V_N^{\rm I}, \qquad \widehat{M}_N = W_N^{{\rm I}\,T} M_N W_N^{\rm I}. \tag{B·12}$$

Thus, for the diagonalization of the mass term in $W_{\rm RHN}$ it is sufficient to flavour rotate N^c:

$$N^c \to W_N^{\rm I}\, N^c. \tag{B·13}$$

This and the two rotations of $\bar{5}_M$ transform the Yukawa term into:

$$e^{i\phi_{10}} N^c \left(U_N^{{\rm I}\,\dagger} W_N^{\rm I}\right)^T \widehat{Y}_N^{\rm I} \left(V_N^{{\rm I}\,\dagger} V_5\right) P_{10}^{(2)}\ \bar{5}_M 5_H, \tag{B·14}$$

which can be recast in the form:

$$e^{i\phi_{\rm I}} N^c \left(P_{\rm I}^a K_{U_N}^{{\rm I}\,T}\right) \widehat{Y}_N^{\rm I} \left(P_{\rm I}^b K_{V_N}^{\rm I}\right) P_{\rm I}\ \bar{5}_M 5_H, \tag{B·15}$$

once the two unitary matrices $(U_N^{{\rm I}\,\dagger} W_N^{\rm I})$ and $(V_N^{{\rm I}\,\dagger} V_5) P_{10}^{(2)}$ are parametrized as in Eq. (B·2).

That is, no reduction of the original eighteen parameters of $Y_N^{\rm I}$ is possible. However, below $M_{\rm GUT}$, the freedom in identifying the SU(2) doublet and singlet leptonic components of $\bar{5}_M$ and 10_M (see Eq. (2·6)) allows the elimination of three phases in the purely leptonic part of this interaction term, say $e^{i\phi_{\rm I}} P_{\rm I}$, and the identification:

$$Y_\nu^{\rm I} = P_I^a K_{U_N}^{I\,T}\, \widehat{Y}_N^{\rm I}\, P_{\rm I}^b K_{V_N}^{\rm I}. \tag{B·16}$$

The situation for the seesaw of type III mirrors exactly that for the seesaw of type I. The matrix $M_{24{\rm M}}$ is diagonalized as M_N, reducing the Yukawa interaction to:

$$e^{i\phi_{\rm III}}\, 5_H\, 24_M \left(P_{\rm III}^a K_{U_N}^{{\rm III}\,T}\right) \widehat{Y}_N^{\rm III} \left(P_{\rm III}^b K_{V_N}^{\rm III}\right) P_{\rm III}\, \bar{5}_M, \tag{B·17}$$

and the connection between $Y_\nu^{\rm III}$ and $\widehat{Y}_N^{\rm III}$ is as in Eq. (B·16), with the replacement ${\rm I} \to {\rm III}$.

For the seesaw of type II, the symmetric matrix $Y_N^{\rm II}$ is diagonalized as Y_S in Eq. (B·1), and no diagonalization is needed for M_{15}, which is just a number. Thus, the first Yukawa interaction in $W_{15{\rm H}}$ reduces to:

$$e^{i2\phi_{\rm II}} \frac{1}{\sqrt{2}} \bar{5}_M P_{\rm II} \left(K_{W_N}^{\rm II} P_{\rm II}^a\right)^T \widehat{Y}_N^{\rm II} \left(K_{W_N}^{\rm II} P_{\rm II}^a\right) P_{\rm II} 15_H \bar{5}_M. \tag{B·18}$$

As before, the phase $\phi_{\rm II}$ and the two in $P_{\rm II}$ can be eliminated in the purely leptonic part of this interaction. The Yukawa coupling of this part, has therefore only nine physical parameters, those in the symmetric matrix $Y_\nu^{\rm II}$, which can be expressed as:

$$Y_\nu^{\rm II} = \left(K_{W_N}^{\rm II} P_{\rm II}^a\right) \widehat{Y}_N^{\rm II} \left(K_{W_N}^{\rm II} P_{\rm II}^a\right)^T. \tag{B·19}$$

Appendix C
—— Evolution of the vevs of the field 24_H ——

We show here that the RGEs in Eq. (2·27) hold in a model-independent way for the leading components of the scalar and auxiliary *vevs* of any chiral superfield ϕ_j, with anomalous dimension γ_j, provided the scalar *vev* of ϕ_j: *i)* exists in the SUSY limit, *i.e.* it is determined by superpotential parameters; *ii)* is much larger than the SUSY-breaking scale $\widetilde{m}$. We call v_j the *vev* of ϕ_j in the SUSY limit , and M the superpotential large scale that determines it.

As usual, we expand the superpotential and Kähler potential in powers of the SUSY-breaking *vev* F_{X} over the cutoff scale, $F_{\mathsf{X}}/M_{\rm cut} = f_{\mathsf{X}}$, which we denote generically by $\widetilde{m}$:

$$\begin{aligned} W(\widetilde{m}) &= W + \widetilde{m}\theta^2\widetilde{W}, \\ K(\widetilde{m}) &= K + [\widetilde{m}\theta^2\widetilde{K} + \text{H.c.}] + \mathcal{O}(\widetilde{m}^2). \end{aligned} \tag{C·1}$$

The scalar potential is then given by

$$V = -F^i K_i^j F_j + [(W^j - \widetilde{m}\widetilde{K}^j)F_j + \widetilde{m}\widetilde{W} + \text{H.c.}] + \mathcal{O}(M^2\widetilde{m}^2), \tag{C·2}$$

where, differently than in the text, we use covariant and contravariant indices. The symbols F_j (F^i) denote the auxiliary components of the chiral (antichiral) multiplets ϕ_j (ϕ^i). In contrast, a subscript (superscript) index in V, W, $\widetilde{W}$, K, and $\widetilde{K}$ denotes the derivative of these functions with respect to ϕ^j (ϕ_i).

From the derivative of the scalar potential with respect to F_j, we find:

$$F^i = (W^j - \widetilde{m}\widetilde{K}^j)(K^{-1})^i_j, \tag{C·3}$$

which allows to recast the scalar potential in the form:

$$V = (W^j - \widetilde{m}\widetilde{K}^j)(K^{-1})^i_j(W_i - \widetilde{m}\widetilde{K}_i) + [\widetilde{m}\widetilde{W} + \text{H.c.}] + \mathcal{O}(M^2\widetilde{m}^2). \tag{C·4}$$

Notice that if it is possible to reduce the Kähler potential to its canonical form, with $\widetilde{K} = 0$ and $K^i_j = \delta^i_j$, then the scalar potential reduces to that of Eq. (2·18) in the specific case of the MSSU(5) model. The condition of minimality of the Kähler potential is, however, not required in the following.

In the SUSY limit $\widetilde{m} \to 0$, the auxiliary component F^j must vanish and therefore:

$$W^j = 0, \tag{C·5}$$

which determines the scalar *vev* v_j independently of the Kähler potential, in general of $\mathcal{O}(M)$.

For nonvanishing $\widetilde{m}$, this scalar *vev* so obtained is shifted by terms of $\mathcal{O}(\widetilde{m})$ and an auxiliary *vev* is also generated. At $\mathcal{O}(M^2\widetilde{m})$, the vacuum condition is:

$$V^k \;=\; W^{jk}(K^{-1})^i_j(W_i - \widetilde{m}\widetilde{K}_i) + \widetilde{m}\widetilde{W}^k + \mathcal{O}(M\widetilde{m}^2) \;=\; 0, \tag{C·6}$$

where we have used the fact that W_i, and therefore also $(W_i - \widetilde{m}\widetilde{K}_i)$, are at most of $\mathcal{O}(M\widetilde{m})$, from which we obtain the leading, nonvanishing contribution to the auxiliary *vev*, of $\mathcal{O}(M\widetilde{m})$:

$$F_j = (K^{-1})^i_j(W_i - \widetilde{m}\widetilde{K}_i) = -\widetilde{m}\frac{\widetilde{W}^k}{W^{jk}} + \mathcal{O}(\widetilde{m}^2). \tag{C·7}$$

In spite of the apparent dependence on the Kähler potential shown by the first equality in this equation, F_j is in reality determined by the superpotential only.

Through the same analysis it is easy to see that the higher order terms δv_j, δF_j and $\delta^2 v_j$ of the scalar and auxiliary *vevs* of ϕ_j expanded in $\widetilde{m}/M$ do depend on the Kähler potential. Thus, their evolution equation is sensitive to the vertex corrections that this potential gets at the quantum level and are, therefore, quite different from those for v_j and F_j.

Having established that v_j and F_j are independent of the Kähler potential, it is then easy to obtain their evolution equations independently of their specific expression. Redefinitions of the field ϕ_i act as:

$$\phi_j \to \exp\left[\left(\gamma_j + \tilde{\gamma}_j\theta^2\right)\, t\right]\, (\phi_j)^r\,, \tag{C·8}$$

with the label r distinguishing the redefined field. Since these redefinitions are just a renaming, we can equate the *vevs* of the left-hand side and right-hand side in this equation, obtaining:

$$\frac{1}{t}\left\{\langle(\phi_j)^r\rangle - \langle\phi_j\rangle\right\} = -\left\{\boldsymbol{\gamma}_j v_j + \theta^2\left[\boldsymbol{\gamma}_j F_j + \widetilde{\boldsymbol{\gamma}}_j v_j\right]\right\} + \mathcal{O}(t), \tag{C·9}$$

which reproduces Eq. (2·27) in the limit $t \to 0$.

Appendix D
—— *Renormalization of effective couplings* ——

We prove in this appendix that the effective couplings evolve all the way up to the cutoff scale following RGEs that are formally those of an MSSU(5) model broken at $\mathcal{O}(s)$, but with gauge interactions respecting the SU(5) symmetry. We prove this specifically for the $\bar{5}_M$ sector with a type I seesaw. We rewrite the superpotential class of operators Op^5 as:

$$Op^5 \;=\; \sqrt{2}\;(\bar{5}_M)_A\left(\boldsymbol{Y}^5(24_H)\right)_{ABCD}\;(10_M)_{BC}\,(\bar{5}_H)_D\,, \tag{D·1}$$

where the effective coupling $\boldsymbol{Y}^5(24_H)$ is defined as:

$$\left(\boldsymbol{Y}^5(24_H)\right)_{ABCD} \;=\; \sum_{n+m=0}^{k} C^5_{n,m}\left(\frac{24_H^T}{M_{\rm cut}}\right)^n_{AB}\left(\frac{24_H}{M_{\rm cut}}\right)^m_{CD}, \tag{D·2}$$

with $(24_H)^0_{AB}$ defined in Eq. (A·23). It will be shown later how the effective couplings $\boldsymbol{Y}^5_i$ ($i = D,E,DU,LQ$) used in Sec. 3.1 are related to this. Similarly, we rewrite $Op^5(\mathsf{X})$ in Eq. (6·3), which gives rise to $\widetilde{O}p^5$, as:

$$Op^5(\mathsf{X}) \;=\; \sqrt{2}\;(\bar{5}_M)_A\left(\widetilde{\boldsymbol{Y}}^5(\mathsf{X},24_H)\right)_{ABCD}\;(10_M)_{BC}\,(\bar{5}_H)_D\,, \tag{D·3}$$

with $(\widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H))_{ABCD}$ defined as

$$\left(\widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H)\right)_{ABCD} = \left(\frac{\mathsf{X}}{M_{\rm cut}}\right) \sum_{n+m=0}^{k} a^5_{n,m} C^5_{n,m} \left(\frac{24_H^T}{M_{\rm cut}}\right)^n_{AB} \left(\frac{24_H}{M_{\rm cut}}\right)^m_{CD}. \qquad \text{(D·4)}$$

We expand $\boldsymbol{Y}^5(24_H)$ around the *vev* of 24_H: $\langle 24_H\rangle = \sqrt{2}\,\langle 24_H^{24}\rangle T^{24} + 24'_H$, with T^{24} the 24th generator of SU(5), and $24'_H$ denoting the quantum fluctuations of the field, including the Nambu-Goldstone modes:

$$\boldsymbol{Y}^5(24_H) = \boldsymbol{Y}^5(\langle 24_H\rangle) + \boldsymbol{Y}^{5\prime}(\langle 24_H\rangle) 24'_H + \mathcal{O}({24'_H}^2), \qquad \text{(D·5)}$$

where we have suppressed here all SU(5) indices. (These indices should be handled with care in the second term, where the prime indicates the derivative with respect to the field 24_H.) We shall reinstate them back when it is important to show the SU(5) structure in quantities containing these couplings.

As discussed in Sec. 2.2, $\langle 24_H^{24}\rangle$ is decomposed in the *vevs* of the scalar and auxiliary component, $\langle B_H\rangle$ and $\langle F_{B_H}\rangle$, as in Eq. (2·10). At the leading order in $\widetilde{m}/M_{\rm GUT}$, which is sufficient for this discussion, it is $\langle B_H\rangle = v_{24}$ and $\langle F_{B_H}\rangle = F_{24}$. Thus, the lowest order in this expansion is

$$\boldsymbol{Y}^5(\langle 24_H\rangle) \equiv \boldsymbol{Y}^5 + \theta^2 \boldsymbol{A}^5_{F_{24}} = \sum_{n+m=0}^{k} C^5_{n,m} \left(\frac{\langle 24_H^{24}\rangle}{M_{\rm cut}}\right)^{n+m} \left(\sqrt{2} T^{24}_{\bar{5}_M}\right)^n \left(\sqrt{2} T^{24}_{\bar{5}_H}\right)^m, \qquad \text{(D·6)}$$

where the lower labels in the generator T^{24} denotes whether it is acting on the representation $\bar{5}_M$ or $\bar{5}_H$. Notice the use of the symbol $\boldsymbol{Y}^5$ for the term in which all *vevs* are scalar, and of $\boldsymbol{A}^5_{F_{24}}$ for the term in which one of them is the auxiliary *vev*. It is justified by the fact that $\boldsymbol{Y}^5$ and $\boldsymbol{A}^5_{F_{24}}$ reduce to the couplings in Eq. (3·6) and to the couplings $\boldsymbol{A}^5_{i,F_{24}}$ discussed in Sec. 6.1, when the generators T^{24} in this equation are replaced by their eigenvalues. Contrary to that of Sec. 3, the formulation given here allows us to maintain an SU(5)-symmetric structure.

Similarly, the lowest order in the expansion of $\widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H)$, with X also replaced by the *vev* of its auxiliary component, is

$$\widetilde{\boldsymbol{Y}}^5(\langle\mathsf{X}\rangle, \langle 24_H\rangle) \equiv \theta^2 \boldsymbol{A}^5|_{F_{\mathsf{X}}} = \theta^2 \sum_{n+m=0}^{k} \widetilde{C}^5_{n,m} \left(\frac{\langle 24_H^{24}\rangle}{M_{\rm cut}}\right)^{n+m} \left(\sqrt{2} T^{24}_{\bar{5}_M}\right)^n \left(\sqrt{2} T^{24}_{\bar{5}_H}\right)^m, \qquad \text{(D·7)}$$

and the complete effective coupling $\boldsymbol{A}^5$ is

$$\boldsymbol{A}^5 = \boldsymbol{A}^5_{F_{\mathsf{X}}} + \boldsymbol{A}^5_{F_{24}}. \qquad \text{(D·8)}$$

We are now in a position to calculate the one-loop corrections to the Kähler potential coming from the above effective couplings. For example, the corrections to $\bar{5}_M \bar{5}_M^*$ due to the effective coupling $\boldsymbol{Y}^5(24_H)$, with the fields 10_M and $\bar{5}_H$ exchanged in the loop, are

$$\begin{aligned} \delta\mathcal{K} \supset -4t\bar{5}_M \Big[& \boldsymbol{Y}^5(24_H) \boldsymbol{Y}^{5\dagger}(24_H) + \widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H) \boldsymbol{Y}^{5\dagger}(24_H) \\ & + \boldsymbol{Y}^5(24_H) \widetilde{\boldsymbol{Y}}^{5\dagger}(\mathsf{X}, 24_H) + \widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H) \widetilde{\boldsymbol{Y}}^{5\dagger}(\mathsf{X}, 24_H) \Big] \bar{5}_M^*, \end{aligned} \qquad \text{(D·9)}$$

where t is the usual factor $t = \ln(Q/Q_0)/(16\pi^2)$. The saturation of SU(5) indices not shown in this equation, is, for example for the first term, as follows

$$\left(\boldsymbol{Y}^5(24_H)\right)_{A'B'C'D'} \frac{1}{2}\left(\delta_{B'B}\delta_{C'C} - \delta_{B'C}\delta_{C'B}\right)\delta_{D'D}\left(\boldsymbol{Y}^{5\,\dagger}(24_H)\right)_{ABCD}. \qquad (\text{D}\cdot 10)$$

In this expression, the indices A and A' are to be contracted with those of $\bar{5}_M$ and $\bar{5}_M^*$, and the factors $(1/2)(\delta_{B'B}\delta_{C'C} - \delta_{B'C}\delta_{C'B})$ and $\delta_{D'D}$ come, respectively, from the propagators of the fields 10_M and $\bar{5}_H$ exchanged in the loop. For vanishing NROs, it is $\boldsymbol{Y}^5(24_H)_{ABCD} = Y^5\delta_{AB}\delta_{CD}$, and the above product reduces to:

$$2\delta_{A'A}Y^5Y^{5\,\dagger}. \qquad (\text{D}\cdot 11)$$

The other terms in Eq. (D·9) for $\delta\mathcal{K}$ have the same SU(5) structure.

The form of $\delta\mathcal{K}$ simplifies considerably if we neglect the quantum fluctuation $24_H'$. In this limit it is easy to see that the corrections to $\bar{5}_M\bar{5}_M^*$ listed above, indeed, decompose in the corrections to the terms $D^cD^{c\,*}$ and L^*L. We show this explicitly for the first term of Eq. (D·9).

When the dynamical part of the field 24_H is neglected, the coupling $\boldsymbol{Y}^5(24_H)$ reduces to that of Eq. (D·6), and the SU(5) indices $ABCD$ are now carried by the generators T^{24}:

$$\left(\sqrt{2}T^{24}_{\bar{5}_M}\right)^n_{AB}\left(\sqrt{2}T^{24}_{\bar{5}_H}\right)^m_{CD} = \left(\sqrt{2}T^{24}_{\bar{5}_M}\right)^n_{AA}\left(\sqrt{2}T^{24}_{\bar{5}_H}\right)^m_{DD}\delta_{AB}\,\delta_{CD}. \qquad (\text{D}\cdot 12)$$

By plugging this expression in Eq. (D·10), we obtain:

$$\sum_{n'+m'=0}^{k}\sum_{n+m=0}^{k} C^{(5)}_{n',m'}C^{(5)\,\dagger}_{n,m}\left(\frac{\langle 24^{24}_H\rangle}{M_{\text{cut}}}\right)^{n'+m'}\left(\frac{\langle 24^{24}_H\rangle}{M_{\text{cut}}}\right)^{n+m}\frac{1}{2}\sum_D\left(\delta_{A'A}\delta_{DD} - \delta_{A'D}\delta_{AD}\right)$$
$$\times\left(\sqrt{2}T^{24}_{\bar{5}_M}\right)^{n'}_{A'A'}\left(\sqrt{2}T^{24}_{\bar{5}_H}\right)^{m'}_{DD}\left(\sqrt{2}T^{24}_{\bar{5}_M}\right)^{n}_{AA}\left(\sqrt{2}T^{24}_{\bar{5}_H}\right)^{m}_{DD}. \qquad (\text{D}\cdot 13)$$

We focus on the first Kronecker δ in parenthesis. The second one, due to the antisymmetry of field 10_M, taken with its minus sign, gives the same result of the first and cancels the factor 1/2 in front of the parenthesis. The original SU(5) indices appearing in Eq. (D·10) are now fixed as follows: $A' = B' = B = A$ and $C' = D' = D = C$. The index A, equal to A' fixes the component of the external fields $\bar{5}_M$ and $\bar{5}_M^*$. The choice of this index corresponds to picking up the corrections to $D^cD^{c\,*}$ and L^*L in the Kähler potential. Similarly, the value of the summed index D fixes the component of the field $\bar{5}_H$ running in the loop: a component of H^C_D when $D \supset \{1,2,3\}$, and a component of H_d when $D \supset \{4,5\}$. The fact that the indices B and C are fixed indicates that the component of the field 10_M in the other propagator of the loop is also automatically fixed by the symmetry. That is, the following choice for A and D: $A \supset \{4,5\}$ and $D \supset \{4,5\}$, but with $A \neq D$, fixes the external fields to be L^* and L and the fields exchanged in the loop H_d and E^c. By choosing D in the set $\{1,2,3\}$, the fields exchanged in the loop are H^C_D and Q. Thus, for each of such choices, the external fields $\bar{5}_M$ and $\bar{5}_M^*$ decompose in their SM components. It becomes, therefore, possible to assign the eigenvalues of $T^{24}_{\bar{5}_M}$ and

$T^{24}_{\bar{5}_H}$ in each diagram, and the corresponding effective couplings get assigned to the two vertices of the loop. The previous product becomes now:

$$\sum_{n'+m'=0}^{k} \sum_{n+m=0}^{k} C^{(5)}_{n',m'} C^{(5)\dagger}_{n,m} \left(\frac{\langle 24^{24}_H\rangle}{M_{\rm cut}}\right)^{n'+m'} \left(\frac{\langle 24^{24}_H\rangle}{M_{\rm cut}}\right)^{n+m} \sum_i \left(I_{\bar{5}_M}\right)^{n'}_i \left(I_{\bar{5}_H}\right)^{m'}_i \left(I_{\bar{5}_M}\right)^{n}_i \left(I_{\bar{5}_H}\right)^{m}_i , \tag{D·14}$$

with the sum over D now replaced by the sum over $i = D, E, DU, LQ$. That is, $\boldsymbol{Y}^5$ indeed splits into the four couplings $\boldsymbol{Y}^5_i$, and, still in the limit in which the dynamical part of the field 24_H is neglected, Eq. (D·9) breaks into an equation with the corrections to the term $D^c D^{c*}$:

$$\delta\mathcal{K} \supset -tD^c \sum_i c^*_{(D^c,i)} \Big[\boldsymbol{Y}^5_i(24_H)\boldsymbol{Y}^{5\,\dagger}_i(24_H) + \widetilde{\boldsymbol{Y}}^5_i(\mathsf{X}, 24_H)\boldsymbol{Y}^{5\,\dagger}_i(24_H) + \boldsymbol{Y}^5_i(24_H)\widetilde{\boldsymbol{Y}}^{5\,\dagger}_i(\mathsf{X}, 24_H) + \widetilde{\boldsymbol{Y}}^5_i(\mathsf{X}, 24_H)\widetilde{\boldsymbol{Y}}^{5\,\dagger}_i(\mathsf{X}, 24_H)\Big] D^{c*}_{,} \tag{D·15}$$

and another one with the corrections to the term L^*L. The group factors $c^*_{(D^c,i)}$ and $c_{(L,i)}$ are in general different for distinct indices i, but reproduce the coefficient 4 in Eq. (D·9) when summed over i, $\sum_i c^*_{(D^c,i)} = \sum_i c_{(L,i)} = 4$.

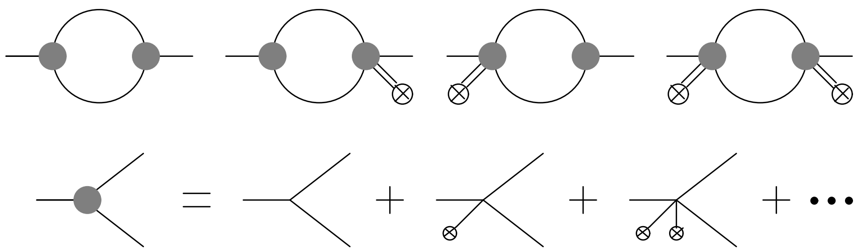

Fig. 4. One-loop diagrams contributing corrections to the Kähler potential and effective vertex: the black circle denotes the effective coupling as shown in the bottom figure. Here a single (double) line terminated with $\otimes$ indicates the *vev* of 24_H (the auxiliary field X).

After the above discussion, it is easy to see that the one-loop diagrams expressed in terms of effective coupling correctly sum up all the one-loop diagrams expressed in terms of the original couplings, as schematically illustrated in Fig. 4. In the figure, the four diagrams in the top correspond, in order, to the four terms in Eq. (D·9).

The part of the corrections in which $24'_H$ is neglected, can be reabsorbed by field redefinitions, as the chiral field 24_H is replaced by its *vevs* (numbers) and an SU(5) generator. This is not true for the contributions to these corrections in which a chiral field 24_H and an antichiral one acquire the auxiliary *vev*. These terms will end up contributing to the effective soft masses squared, as usual. This procedure, repeated for all fields, defines all anomalous dimensions in terms of the effective couplings. Thus, suppressing SU(5) indices, we can express $\boldsymbol{\gamma}_{\bar{5}_M}$ as:

$$\boldsymbol{\gamma}_{\bar{5}_M} = \frac{1}{2}\Big\{\big[\text{gauge contr.}\big] + 4\boldsymbol{Y}^{5\,*}\boldsymbol{Y}^{5\,T} + \cdots\Big\} + \boldsymbol{\zeta}_{\gamma_{\bar{5}_M}}, \tag{D·16}$$

where in the dots are included the contributions from all other effective Yukawa couplings, whereas all remaining quantities are collected in $\boldsymbol{\zeta}_{\gamma_{\bar{5}_M}}$. Notice that, in the limit of vanishing NROs, the product of effective couplings in this anomalous dimension reduces to the expression in Eq. (D·11) and we recover the numerical coefficient in the anomalous dimension $\gamma_{\bar{5}_M}$ of Eq. (E·63). The corresponding quantity $\tilde{\gamma}_{\bar{5}_M}$ is

$$\widetilde{\boldsymbol{\gamma}}_{\bar{5}_M} = \Big\{ -[\text{gaugino contr.}] + 4\boldsymbol{Y}^{5\,*}\boldsymbol{A}^{5\,T} + \cdots \Big\} + \widetilde{\boldsymbol{\zeta}}_{\gamma_{\bar{5}_M}}, \tag{D·17}$$

with $\widetilde{\boldsymbol{\zeta}}_{\gamma_{\bar{5}_M}}$ playing the same role that $\boldsymbol{\zeta}_{\gamma_{\bar{5}_M}}$ has for $\boldsymbol{\gamma}_{\bar{5}_M}$. Notice that it is truly the complete effective coupling $\boldsymbol{A}^5$ that appears here, to which both, $\boldsymbol{Y}^5(24_H)$ and $\widetilde{\boldsymbol{Y}}^5(\mathsf{X}, 24_H)$, contribute. The minus sign in front of the [gaugino contr.] is just a symbolic reminder of the fact that this term has opposite sign to the term [gauge contr.] in the definition of $\boldsymbol{\gamma}_{\bar{5}_M}$.

We remind that the terms expressed in term of effective couplings in $\boldsymbol{\gamma}_{\bar{5}_M}$ and $\widetilde{\boldsymbol{\gamma}}_{\bar{5}_M}$ are just numbers (with generators) and do not consist of any dynamical field. This is not true for the quantities $\boldsymbol{\zeta}_{\gamma_{\bar{5}_M}}$ and $\widetilde{\boldsymbol{\zeta}}_{\gamma_{\bar{5}_M}}$, which collect all the contributions that cannot be expressed in terms of effective couplings, in particular, those discussed in Sec. 10.1. As already remarked there, these quantities do break the picture of effective couplings. They are not, in general, subleading with respect to the other terms in $\boldsymbol{\gamma}_{\bar{5}_M}$ and $\widetilde{\boldsymbol{\gamma}}_{\bar{5}_M}$, but they give rise to subleading contributions to sFVs. Thus, we neglect them in the following discussion.

Field redefinitions through the just defined anomalous dimension, such as $\bar{5}_M \to \exp\,[(\boldsymbol{\gamma}_{\bar{5}_M} + \tilde{\boldsymbol{\gamma}}_{\bar{5}_M}\theta^2)\,t]\,\bar{5}_M$, and similar ones for all the other fields, allow to recover the minimality of the Kähler potential, up to soft masses squared (and the contributions collected in $\boldsymbol{\zeta}_{\gamma_{\bar{5}_M}}$ and $\widetilde{\boldsymbol{\zeta}}_{\gamma_{\bar{5}_M}}$), by shifting the couplings in the superpotential. It is then very easy to express the one-loop leading logarithm modifications of the original couplings in the superpotential, such as $C^5_{n,m}$, in terms of these anomalous dimensions. To simplify further our notation, we absorb also the group theoretical factors into the original superpotential coefficients:

$$\boldsymbol{Y}^5_{ABCD} \;=\; \sum_{n=0}^{k} (C^5_n)_{ABCD} \left(\frac{\langle B_H\rangle}{M_{\text{cut}}}\right)^n. \tag{D·18}$$

Similarly, $\boldsymbol{A}^5_{F_\mathsf{X}}$ and $\boldsymbol{A}^5_{F_{24}}$ have now the simplified form:

$$\begin{aligned}
(\boldsymbol{A}^5_{F_\mathsf{X}})_{ABCD} &= \sum_{n=0}^{k} \left(\widetilde{C}^5_n\right)_{ABCD} \left(\frac{\langle B_H\rangle}{M_{\text{cut}}}\right)^n, \\
(\boldsymbol{A}^5_{F_{24}})_{ABCD} &= \sum_{n=0}^{k} n f_{24}\,(C^5_n)_{ABCD} \left(\frac{\langle B_H\rangle}{M_{\text{cut}}}\right)^n.
\end{aligned} \tag{D·19}$$

By keeping into account the evolution of the *vevs* of the field 24_H, it is easy to obtain the RGEs for the effective Yukawa and trilinear couplings:

$$\left(\dot{\boldsymbol{Y}^5\big|_n}\right) = \Big\{ \boldsymbol{\gamma}^T_{\bar{5}_M}\,\boldsymbol{Y}^5\big|_n + \boldsymbol{Y}^5\big|_n \boldsymbol{\gamma}_{10_M} + \boldsymbol{\gamma}_{\bar{5}_H}\boldsymbol{Y}^5\big|_n \Big\},$$

$$
\begin{aligned}
(\dot{\boldsymbol{A}^5}\big|_n) = & \left\{\boldsymbol{\gamma}^T_{\bar{5}_M}\boldsymbol{A}^5\big|_n + \boldsymbol{A}^5\big|_n\gamma_{10_M} + \gamma_{\bar{5}_H}\boldsymbol{A}^5\big|_n\right\} \\
& + \left\{\widetilde{\boldsymbol{\gamma}}^T_{\bar{5}_M}\boldsymbol{Y}^5\big|_n + \boldsymbol{Y}^5\big|_n\widetilde{\gamma}_{10_M} + \widetilde{\gamma}_{\bar{5}_H}\boldsymbol{Y}^5\big|_n\right\}, \qquad \text{(D·20)}
\end{aligned}
$$

where we have suppressed the SU(5) indices, for simplicity. The one-loop evolution equation for the coefficients C_n^5 and $\widetilde{C}_n^5$ needed to derive them are discussed in detail in Sec. 10.2. The evolution equation for $(\boldsymbol{A}^5_{F\mathsf{X}})|_n$ and that for $(\boldsymbol{A}^5_{F_{24}})|_n$ are formally like the equation for $\boldsymbol{A}^5|_n$, except for one spurious term. This is $n\widetilde{\boldsymbol{\gamma}}_{24_H}$, in the RGEs for $(\boldsymbol{A}^5_{F\mathsf{X}})|_n$, and $-n\widetilde{\boldsymbol{\gamma}}_{24_H}$, in that for $(\boldsymbol{A}^5_{F_{24}})|_n$. Thus, the role played by $\boldsymbol{A}^5_{F_{24}}$ is very important for the correct evolution of $\boldsymbol{A}^5\big|_n$. The RGEs for the effective couplings $\boldsymbol{Y}^5$ and $\boldsymbol{A}^5$, are obtained from $\boldsymbol{Y}^5|_n$ and $\boldsymbol{A}^5|_n$ with a summation over n.

Having proven already how the products of the SU(5)-symmetric effective couplings introduced in this section correctly decompose into products of the decomposed effective couplings of Sec. 3, we can split the obtained RGEs for SU(5) multiplets into RGEs for their SM components. For example, the RGEs for the four effective couplings $\boldsymbol{Y}_i^5$ are then

$$
\begin{aligned}
\dot{\boldsymbol{Y}}^5_D &= \boldsymbol{\gamma}^T_{D^c}\boldsymbol{Y}^5_D \;+ \boldsymbol{Y}^5_D\boldsymbol{\gamma}_Q \;+ \boldsymbol{\gamma}_{H_d}\boldsymbol{Y}^5_D, \\
\dot{\boldsymbol{Y}}^5_E &= \boldsymbol{\gamma}^T_L\boldsymbol{Y}^5_E \;+ \boldsymbol{Y}^5_E\boldsymbol{\gamma}_{E^c} \;+ \boldsymbol{\gamma}_{H_d}\boldsymbol{Y}^5_E, \\
\dot{\boldsymbol{Y}}^5_{DU} &= \boldsymbol{\gamma}^T_{D^c}\boldsymbol{Y}^5_{DU} + \boldsymbol{Y}^5_{DU}\boldsymbol{\gamma}_{U^c} + \boldsymbol{\gamma}_{H^C_D}\boldsymbol{Y}^5_{DU}, \\
\dot{\boldsymbol{Y}}^5_{LQ} &= \boldsymbol{\gamma}^T_L\boldsymbol{Y}^5_{LQ} + \boldsymbol{Y}^5_{LQ}\boldsymbol{\gamma}_Q + \boldsymbol{\gamma}_{H^C_D}\boldsymbol{Y}^5_{LQ}, \qquad \text{(D·21)}
\end{aligned}
$$

where the decompositions $\gamma_{\bar{5}_M} \to \{\boldsymbol{\gamma}_{D^c},\boldsymbol{\gamma}_L\}$, $\gamma_{10_M} \to \{\boldsymbol{\gamma}_{U^c},\boldsymbol{\gamma}_Q,\boldsymbol{\gamma}_{E^c}\}$, and $\gamma_{\bar{5}_H} \to \{\boldsymbol{\gamma}_{H^C_D},\boldsymbol{\gamma}_{H_d}\}$ are also made. Among the decomposed anomalous dimensions, $\boldsymbol{\gamma}_L$ and $\boldsymbol{\gamma}_{D^c}$, taken here as example, are given by

$$
\begin{aligned}
\boldsymbol{\gamma}_L &= -2\left(\frac{12}{5}g_5^2\right)\boldsymbol{1} + \boldsymbol{Y}^{5*}_E\boldsymbol{Y}^{5\,T}_E + 3\boldsymbol{Y}^{5*}_{LQ}\boldsymbol{Y}^{5\,T}_{LQ} + \boldsymbol{Y}^{\mathrm{I}\dagger}_N\boldsymbol{Y}^{\mathrm{I}}_N, \\
\boldsymbol{\gamma}_{D^c} &= -2\left(\frac{12}{5}g_5^2\right)\boldsymbol{1} + 2\boldsymbol{Y}^{5*}_D\boldsymbol{Y}^{5\,T}_D + 2\boldsymbol{Y}^{5*}_{DU}\boldsymbol{Y}^{5\,T}_{DU} + \boldsymbol{Y}^{\mathrm{I}\dagger}_{ND}\boldsymbol{Y}^{\mathrm{I}}_{ND}, \qquad \text{(D·22)}
\end{aligned}
$$

and we refer the reader to Appendix E.4 for the remaining ones.

Similarly, $\boldsymbol{A}^5_D$ among the four effective trilinear couplings evolves according to:

$$
\dot{\boldsymbol{A}}^5_D \;=\; \left(\boldsymbol{\gamma}^T_{D^c}\boldsymbol{A}^5_D + \boldsymbol{A}^5_D\boldsymbol{\gamma}_Q + \boldsymbol{\gamma}_{H_d}\boldsymbol{A}^5_D\right) + \left(\widetilde{\boldsymbol{\gamma}}^T_{D^c}\boldsymbol{Y}^5_D + \boldsymbol{Y}^5_D\widetilde{\boldsymbol{\gamma}}_Q + \widetilde{\boldsymbol{\gamma}}_{H_d}\boldsymbol{Y}^5_D\right), \qquad \text{(D·23)}
$$

with the quantities $\widetilde{\boldsymbol{\gamma}}_i$ are also decomposed: $\widetilde{\gamma}_{\bar{5}_M} \to \{\widetilde{\boldsymbol{\gamma}}_{D^c},\widetilde{\boldsymbol{\gamma}}_L\}$, $\widetilde{\gamma}_{\bar{5}_H} \to \{\widetilde{\boldsymbol{\gamma}}_{H^C_D},\widetilde{\boldsymbol{\gamma}}_{H_d}\}$, and $\widetilde{\gamma}_{10_M} \to \{\widetilde{\boldsymbol{\gamma}}_{U^c},\widetilde{\boldsymbol{\gamma}}_Q,\widetilde{\boldsymbol{\gamma}}_{E^c}\}$.

The RGEs listed above are those of a renormalizable model in which only the gauge interactions respect the SU(5) symmetry, whereas all other couplings break it at $\mathcal{O}(s)$.

It is simple to prove that also the effective soft masses squared obey RGEs analogous to those for soft masses squared in this broken MSSU(5) model. A key role in this proof is played, again, by the fact that the wave function renormalization

of the field 24_H is cancelled by the evolution of the 24_H *vevs*. In particular, the RGE for $\widetilde{m}^2_{\bar{5}_M}$ is now replaced by those for $\widetilde{\boldsymbol{m}}^2_L$ and $\widetilde{\boldsymbol{m}}^2_{D^c}$:

$$\begin{aligned}
\dot{\widetilde{\boldsymbol{m}}}^2_L = -8\left(\frac{12}{5}g_5^2 M_G^2\right)\mathbf{1} &+ \ \mathcal{F}(\boldsymbol{Y}^{5\,*}_E, \widetilde{\boldsymbol{m}}^2_L, \widetilde{\boldsymbol{m}}^2_{E^c}, \widetilde{\boldsymbol{m}}^2_{H_d}, \boldsymbol{A}^{5\,*}_E) \\
&+ 3\mathcal{F}(\boldsymbol{Y}^{5\,*}_{LQ}, \widetilde{\boldsymbol{m}}^2_L, \widetilde{\boldsymbol{m}}^{2\,*}_Q, \widetilde{\boldsymbol{m}}^2_{H^C_D}, \boldsymbol{A}^{5\,*}_{LQ}) \\
&+ \ \mathcal{F}(\boldsymbol{Y}^{\mathrm{I}\dagger}_N, \widetilde{\boldsymbol{m}}^2_L, \widetilde{\boldsymbol{m}}^2_{N^c}, \widetilde{\boldsymbol{m}}^2_{H_u}, \boldsymbol{A}^{\mathrm{I}\dagger}_N), \\
\dot{\widetilde{\boldsymbol{m}}}^2_{D^c} = -8\left(\frac{12}{5}g_5^2 M_G^2\right)\mathbf{1} &+ 2\mathcal{F}(\boldsymbol{Y}^5_D, \widetilde{\boldsymbol{m}}^2_{D^c}, \widetilde{\boldsymbol{m}}^2_Q, \widetilde{\boldsymbol{m}}^2_{H_d}, \boldsymbol{A}^5_D) \\
&+ 2\mathcal{F}(\boldsymbol{Y}^5_{DU}, \widetilde{\boldsymbol{m}}^2_{D^c}, \widetilde{\boldsymbol{m}}^{2\,*}_{U^c}, \widetilde{\boldsymbol{m}}^2_{H^C_D}, \boldsymbol{A}^5_{DU}) \\
&+ \ \mathcal{F}(\boldsymbol{Y}^{\mathrm{I}T}_{ND}, \widetilde{\boldsymbol{m}}^2_{D^c}, \widetilde{\boldsymbol{m}}^{2\,*}_{N^c}, \widetilde{\boldsymbol{m}}^2_{H^C_U}, \boldsymbol{A}^{\mathrm{I}T}_{ND}),
\end{aligned} \qquad \text{(D·24)}$$

where $\widetilde{m}^2_{\bar{5}_H}$ and $\widetilde{m}^2_{5_H}$ were also replaced by $\widetilde{\boldsymbol{m}}^2_{H^C_D}, \widetilde{\boldsymbol{m}}^2_{H_d}$ and $\widetilde{\boldsymbol{m}}^2_{H^C_U}, \widetilde{\boldsymbol{m}}^2_{H_u}$, respectively, and $\widetilde{m}^2_{10_M}$ by $\widetilde{\boldsymbol{m}}^2_{U^c}$, $\widetilde{\boldsymbol{m}}^2_Q$, and $\widetilde{\boldsymbol{m}}^2_{E^c}$.

A complete list of all other RGEs for effective couplings and explicit definitions of the various $\boldsymbol{\gamma}_i$ and $\widetilde{\boldsymbol{\gamma}}_i$ can be found in Appendix E.4.

Appendix E

—— *MSSU(5) and nrMSSU(5) models with seesaw: RGEs* ——

We collect in this appendix the one-loop RGEs for the MSSU(5) model and for the nrMSSU(5) models with seesaw sectors, for different values of the scale Q from $M_{\rm weak}$ to $M_{\rm cut}$. We refer the reader to Refs. 66)–69), and 70), 71) for ingredients useful to derive them. The number of RGEs to be listed is very large. Thus, we omit some of these equations, giving algorithms to obtain them from other equations, which we list, or from beta function coefficients and/or anomalous dimensions.[68), 72)] We illustrate in the following how to obtain the omitted RGEs.

- Gauge and gaugino RGEs:
  It is well known that for a collection of gauge groups G_i with gauge couplings g_i, and beta function coefficients b_i, the gauge couplings and the gaugino masses M_i satisfy the evolution equations:

$$\begin{aligned}
\dot{g}_i &= b_i\, g_i^3, \\
\dot{M}_i &= 2b_i\, g_i^2 M_i.
\end{aligned} \qquad \text{(E·1)}$$

  Thus, we omit these RGEs, giving only the beta function coefficient for each specific gauge group.
- Trilinear soft parameters:
  We omit also the RGEs for all couplings of trilinear soft terms since they can be obtained from those of the corresponding Yukawa couplings as follows. We

assume here a Yukawa interaction SY_xTV, where S and T are two generic fields with flavour, V a flavourless one, *i.e.* a Higgs field. Given the RGE for Y_x:

$$\dot{Y}_x = \gamma_S^T Y_x + Y_x \gamma_T + \gamma_V Y_x, \tag{E·2}$$

with γ_S, γ_T, and γ_V the anomalous dimension of the fields S, T, V, the RGE for the corresponding A_x in the soft term $\widetilde{S} A_x \widetilde{T} V$ is

$$\dot{A}_x = \left(\gamma_S^T A_x + A_x \gamma_T + \gamma_V A_x\right) + \left(\tilde{\gamma}_S^T Y_x + Y_x \tilde{\gamma}_T + \tilde{\gamma}_V Y_x\right). \tag{E·3}$$

The quantities $\tilde{\gamma}_S$, $\tilde{\gamma}_T$, and $\tilde{\gamma}_V$ can be built from the corresponding anomalous dimensions γ_S, γ_T, and γ_V with the replacements:

$$g_i^2 \to -2 g_i^2 M_i, \qquad Y_x^\dagger Y_x \to 2 Y_x^\dagger A_x, \qquad Y_x^* Y_x^T \to 2 Y_x^* A_x^T. \tag{E·4}$$

In short:

$$\dot{A}_x = \dot{Y}_x|_{Y_x \to A_x} + \dot{Y}_x|_{\gamma_j \to \tilde{\gamma}_j} \qquad (j = S, T, V). \tag{E·5}$$

- Bilinear soft parameters:
  A similar algorithm can be used to obtain the RGE of the coupling B of a bilinear soft term $\widetilde{S} B \widetilde{T}$ from that of the coupling M of the corresponding superpotential term SMT:

  $$\dot{M} = \gamma_S^T M + M \gamma_T. \tag{E·6}$$

  The two fields S and T are assumed here to have flavour, but they may also be flavourless. The RGE for B is

  $$\dot{B} = \dot{M}|_{M \to B} + \dot{M}|_{\gamma_j \to \tilde{\gamma}_j} \qquad (j = S, T). \tag{E·7}$$

  Thus, we omit also all the RGEs of bilinear soft couplings.
- Soft sfermion and Higgs masses:
  It is also possible to deduce the RGEs for the soft masses of sfermion and Higgs fields from the expressions of their anomalous dimensions, by following the algorithm given hereafter.
  The anomalous dimension of any field, say for example the generic field with flavour T introduced above, gets, in general, contributions from gauge interactions as well as Yukawa interactions (in our specific case, SY_xTV):

  $$\gamma_T = \gamma_{G,T} + \gamma_{Y,T}, \tag{E·8}$$

  with a similar splitting holding also for γ_S and γ_V. The two contributions $\gamma_{G,T}$ and $\gamma_{Y,T}$ are

  $$\gamma_{G,T} = \sum_i c_T^i g_i^2, \qquad \gamma_{Y,T} = c_T^x Y_x^\dagger Y_x, \tag{E·9}$$

  where c_T^i and c_T^x are group theoretical factors. For the fields S and V, the Yukawa-coupling induced $\gamma_{Y,S}$ and $\gamma_{Y,V}$ are

  $$\gamma_{Y,S} = c_S^x Y_x^* Y_x^T, \qquad \gamma_{Y,V} = c_V^x \mathrm{Tr} Y_x^\dagger Y_x = c_V^x \mathrm{Tr} Y_x^* Y_x^T. \tag{E·10}$$

The RGE for the soft mass of the field T, $\widetilde{m}_T^2$, as well as those for $\widetilde{m}_S^2$ and $\widetilde{m}_V^2$, can also be split in two parts:

$$\dot{\widetilde{m}}_T^2 = M_{G,T}^2 + M_{Y,T}^2. \tag{E·11}$$

Notice that $\widetilde{m}_T^2$ and $\widetilde{m}_S^2$ are hermitian matrices, whereas $\widetilde{m}_V^2$, like all soft masses of Higgs fields, is a real number.
The terms $M_{G,T}^2$, $M_{G,S}^2$, and $M_{G,V}^2$, are obtained from $\gamma_{G,T}$, $\gamma_{G,S}$, and $\gamma_{G,V}$, with the replacements:

$$g_i^2 \to 4g_i^2 M_i^2. \tag{E·12}$$

The gaugino masses are assumed here to be real, otherwise the correct replacements would be $4g_i^2|M_i|^2$.
As for the terms $M_{Y,T}^2$, $M_{Y,S}^2$, and $M_{Y,V}^2$, we start assuming that the soft mass terms for the fields T, S and V are of type

$$\widetilde{T}^*\widetilde{m}_T^2\widetilde{T}, \qquad \widetilde{S}^*\widetilde{m}_S^2\widetilde{S}, \qquad \widetilde{V}^*\widetilde{m}_V^2\widetilde{V}. \tag{E·13}$$

Then, $M_{Y,T}^2$, $M_{Y,S}^2$, and $M_{Y,V}^2$, are obtained from $\gamma_{Y,T}$, $\gamma_{Y,S}$, and $\gamma_{Y,V}$, as follows. For the two fields with flavour, T and S, we have:

$$\begin{aligned}
\gamma_{Y,T} &= c_T^x\, Y_x^\dagger Y_x \;\to c_T^x\, \mathcal{F}(Y_x^\dagger, \widetilde{m}_T^2, \widetilde{m}_S^{2\,*}, \widetilde{m}_V^2, A_x^\dagger) = M_{Y,T}^2, \\
\gamma_{Y,S} &= c_S^x\, Y_x^* Y_x^T \to\; c_S^x\, \mathcal{F}(Y_x^*, \widetilde{m}_S^2, \widetilde{m}_T^{2\,*}, \widetilde{m}_V^2, A_x^*) = M_{Y,S}^2,
\end{aligned} \tag{E·14}$$

with the function $\mathcal{F}$ defined as:

$$\begin{aligned}
\mathcal{F}(Y_h^\dagger, \widetilde{m}_j^2, \widetilde{m}_k^{2\,*}, \widetilde{m}_l^2, A_h^\dagger) &= Y_h^\dagger Y_h \widetilde{m}_j^2 + \widetilde{m}_j^2 Y_h^\dagger Y_h \\
&\quad +2\left(Y_h^\dagger \widetilde{m}_k^{2\,*} Y_h + \widetilde{m}_l^2 Y_h^\dagger Y_h + A_h^\dagger A_h\right),
\end{aligned} \tag{E·15}$$

where Y_h is any of the Yukawa couplings in the superpotential, $\widetilde{m}_j^2$ is the mass squared for which the RGE in question is derived, $\widetilde{m}_k^2$ and $\widetilde{m}_l^2$ are the masses squared of the particles exchanged in the loops that induce the RGE, and A_h is the soft counterpart of Y_h. As for the order in which $\widetilde{m}_k^2$ and $\widetilde{m}_l^2$ must appear in $\mathcal{F}$, the following rules apply. For flavour-dependent interactions, and if $\widetilde{m}_j^2$ is the mass squared of a field with flavour, as in this case, then, in second position there must be the conjugate of the mass squared of the other field with flavour, in third, that of the Higgs field. Moreover, it goes without saying that, here and in the following, when the Yukawa coupling on which $\mathcal{F}$ depends is Y_h^*, Y_h^T, or Y_h, instead of $Y_h^\dagger$, the sequences $Y_h^\dagger \cdots Y_h$ in the terms on the right-hand side of Eq. (E·15) must be replaced by $Y_h^* \cdots Y_h^T$, $Y_h^T \cdots Y_h^*$, and $Y_h \cdots Y_h^\dagger$, respectively.
For the flavourless field V, both, the anomalous dimension and the soft mass are real numbers, and the replacement of:

$$\gamma_{Y,V} = c_V^x\, \mathrm{Tr} Y_x^\dagger Y_x \;=\; c_V^x\, \mathrm{Tr} Y_x^* Y_x^T \;=\; c_V^x\, \mathrm{Tr} Y_x^T Y_x^* \;=\; c_V^x\, \mathrm{Tr} Y_x Y_x^\dagger, \tag{E·16}$$

is any of the following terms:

$$\begin{aligned}
&c_V^x\, \mathrm{Tr}\mathcal{F}(Y_x^\dagger, \widetilde{m}_V^2, \widetilde{m}_S^{2\,*}, \widetilde{m}_T^2, A_x^\dagger) = c_V^x\, \mathrm{Tr}\mathcal{F}(Y_x^*, \widetilde{m}_V^2, \widetilde{m}_T^{2\,*}, \widetilde{m}_S^2, A_x^*) \\
&= c_V^x\, \mathrm{Tr}\mathcal{F}(Y_x^T, \widetilde{m}_V^2, \widetilde{m}_S^2, \widetilde{m}_T^{2\,*}, A_x^T) = c_V^x\, \mathrm{Tr}\mathcal{F}(Y_x, \widetilde{m}_V^2, \widetilde{m}_T^2, \widetilde{m}_S^{2\,*}, A_x) \\
&= M_{Y,V}^2.
\end{aligned} \tag{E·17}$$

In this case, the order in which the other two masses $\widetilde{m}_T^2$ and $\widetilde{m}_S^2$ appear in the function $\mathcal{F}$, as well as well whether it is the second of the third mass the one which is chosen to be conjugated is not important, provided the Yukawa coupling and the corresponding trilinear coupling are modified according to Eq. (E·17). This is because of the presence of a Trace in front of $\mathcal{F}$.
Notice that if the Yukawa interaction is among flavourless fields, Y_h is simply a number, the three masses in $\mathcal{F}$ are real numbers, and the order in which these three masses appear is totally irrelevant.
It is possible that the soft mass for the field T is of type $\widetilde{T}\widetilde{m}_T^2\widetilde{T}^*$, whereas those for S and V are as before:

$$\widetilde{T}\widetilde{m}_T^2\widetilde{T}^*, \qquad \widetilde{S}^*\widetilde{m}_S^2\widetilde{S}, \qquad \widetilde{V}^*\widetilde{m}_V^2\widetilde{V}. \tag{E·18}$$

Since $\widetilde{m}_T^2$ is Hermitian, it is $\widetilde{T}\widetilde{m}_T^2\widetilde{T}^* = \widetilde{T}^*\widetilde{m}_T^{2*}\widetilde{T}$. Thus, the RGEs for $\widetilde{m}_S^2$ and $\widetilde{m}_V^2$ are as those described before, with the replacement $\widetilde{m}_T^2 \leftrightarrow \widetilde{m}_T^{2*}$ in $M_{Y,S}^2$ and $M_{Y,V}^2$. In the case of the RGE of $\widetilde{m}_T^2$ itself, the same replacement $\widetilde{m}_T^2 \leftrightarrow \widetilde{m}_T^{2*}$ has to be made. In addition an overall conjugation of $M_{Y,T}^2$ is also needed. In other words, the replacement of $\gamma_{Y,V}$ in Eq. (E·16) is any of the following terms:

$$\begin{aligned} c_V^x\,\mathrm{Tr}\mathcal{F}(Y_x^\dagger, \widetilde{m}_V^2, \widetilde{m}_S^{2*}, \widetilde{m}_T^{2*}, A_x^\dagger) &= c_V^x\,\mathrm{Tr}\mathcal{F}(Y_x^*, \widetilde{m}_V^2, \widetilde{m}_T^2, \widetilde{m}_S^2, A_x^*) \\ = c_V^x\,\mathrm{Tr}\mathcal{F}(Y_x^T, \widetilde{m}_V^2, \widetilde{m}_S^2, \widetilde{m}_T^2, A_x^T) &= c_V^x\,\mathrm{Tr}\mathcal{F}(Y_x, \widetilde{m}_V^2, \widetilde{m}_T^{2*}, \widetilde{m}_S^{2*}, A_x) \\ &= M_{Y,V}^2, \end{aligned} \tag{E·19}$$

whereas the replacements of the terms $\gamma_{Y,T}$ and $\gamma_{Y,S}$ are

$$\begin{aligned} \gamma_{Y,T} &= c_T^x\,Y_x^\dagger Y_x \;\to c_T^x\,\mathcal{F}(Y_x^T, \widetilde{m}_T^2, \widetilde{m}_S^2, \widetilde{m}_V^2, A_x^T) = M_{Y,T}^2, \\ \gamma_{Y,S} &= c_S^x\,Y_x^* Y_x^T \to c_S^x\,\mathcal{F}(Y_x^*, \widetilde{m}_S^2, \widetilde{m}_T^2, \widetilde{m}_V^2, A_x^*) = M_{Y,S}^2. \end{aligned} \tag{E·20}$$

In summary, a Yukawa coupling Y (and the corresponding coupling A) can appear in the function $\mathcal{F}$ as Y, Y^*, $Y^\dagger$, Y^T. The fact that the mass of a field with flavour has a conjugate or not in $\mathcal{F}$ depends on the type of interaction of this field, *i.e.* in which way it contributes to its anomalous dimension, and on the type of soft term this field has.
We illustrate the above sets of rules with two examples. We start with the SU(2) doublet Q of the MSSM, which has anomalous dimension:

$$\gamma_Q = -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{60}g_1^2\right)\mathbf{1} + Y_U^\dagger Y_U + Y_D^\dagger Y_D, \tag{E·21}$$

where g_1, g_2, and g_3 are the three MSSM gauge couplings. The RGE for $\widetilde{m}_Q^2$ is

$$\dot{\widetilde{m}}_Q^2 = M_{G,Q}^2 + \mathcal{F}(Y_U^\dagger, \widetilde{m}_Q^2, \widetilde{m}_{U^c}^2, \widetilde{m}_{H_u}^2, A_U^\dagger) + \mathcal{F}(Y_D^\dagger, \widetilde{m}_Q^2, \widetilde{m}_{D^c}^2, \widetilde{m}_{H_d}^2, A_D^\dagger), \tag{E·22}$$

where $M_{G,Q}^2$ is given by

$$M_{G,Q}^2 = -8\left(\frac{4}{3}g_3^2 M_3^2 + \frac{3}{4}g_2^2 M_2^2 + \frac{1}{60}g_1^2 M_1^2\right)\mathbf{1}. \tag{E·23}$$

In the argument of the two functions $\mathcal{F}$, $\widetilde{m}^2_{U^c}$ and $\widetilde{m}^2_{D^c}$ are not conjugated because we have defined the soft mass terms of the SU(2) singlets U^c, D^c, and E^c as:

$$\widetilde{U}^c \widetilde{m}^2_{U^c} \widetilde{U}^{c\,*}, \quad \widetilde{D}^c \widetilde{m}^2_{D^c} \widetilde{D}^{c\,*}, \quad \widetilde{E}^c \widetilde{m}^2_{E^c} \widetilde{E}^{c\,*}, \tag{E·24}$$

i.e. differently than those for the doublets, Q and L, which are

$$\widetilde{Q}^* \widetilde{m}^2_Q \widetilde{Q}, \quad \widetilde{L}^* \widetilde{m}^2_L \widetilde{L}. \tag{E·25}$$

With this definition, the mass parameters entering in the 6×6 sfermion mass matrices, conventionally written in the basis of the superpartners of left- ($\widetilde{u}_L = \widetilde{U}$, $\widetilde{d}_L = \widetilde{D}$, $\widetilde{e}_L = \widetilde{E}$), and right-handed fermions ($\widetilde{u}_R = \widetilde{U}^c$, $\widetilde{d}_R = \widetilde{D}^c$, $\widetilde{e}_R = \widetilde{E}^c$) are $\widetilde{m}^2_Q$ and $\widetilde{m}^2_{U^c}$ for up-squarks, $\widetilde{m}^2_Q$ and $\widetilde{m}^2_{D^c}$ for down-squarks, and $\widetilde{m}^2_L$ and $\widetilde{m}^2_{E^c}$ for sleptons. In contrast, the choices $\widetilde{\bar{5}}_M \widetilde{m}^2_{5_M} \widetilde{\bar{5}}^*_M$, $\widetilde{N^c} \widetilde{m}^2_{N^c} \widetilde{N^c}^*$ and $\widetilde{24}_M \widetilde{m}^2_{24_M} \widetilde{24}^*_M$ made in the soft SUSY-breaking potentials for the MSSU(5) model with a seesaw of type I and III, are purely conventional and motivated mainly by aesthetic reasons. That is, with the above choice for $\widetilde{m}^2_{5_M}$, the field D^c has the same type of soft mass before and after the breaking of SU(5).
The second example we give here is that of an SU(2) singlet of the MSSM, U^c, with anomalous dimension:

$$\gamma_{U^c} = -2\left(\frac{4}{3}g_3^2 + \frac{4}{15}g_1^2\right)\mathbf{1} + 2Y_U^* Y_U^T. \tag{E·26}$$

The corresponding RGE for $\widetilde{m}^2_{U^c}$, defined in Eq. (E·24), is given by

$$\dot{\widetilde{m}}^2_{U^c} = M^2_{G,U^c} + 2\mathcal{F}(Y_U, \widetilde{m}^2_{U^c}, \widetilde{m}^2_Q, \widetilde{m}^2_{H_u}, A_U), \tag{E·27}$$

with obvious definition of M^2_{G,U^c}.
The algorithm presented here is sufficient to deduce the RGEs for the soft masses of all scalar fields from the expression of their anomalous dimensions. As in the case of the bilinear and trilinear soft parameters, also these RGEs could be omitted. Nevertheless, since this algorithm is a little involved, we list them as in Eq. (E·22) or Eq. (E·27), with a further abbreviation of our notation, which consists in writing $\mathcal{F}(Y_h^\dagger, \widetilde{m}^2_j, \widetilde{m}^2_k, \widetilde{m}^2_l, A_h^\dagger)$ as $\mathcal{F}_{(Y_h^\dagger, \widetilde{j}, \widetilde{k}, \widetilde{l}, A_h^\dagger)}$.

E.1. $Q < M_{\rm ssw}$

When $Q < M_{\rm ssw}$, the typical scale of the heavy fields realizing the seesaw mechanism, the superpotential is that of the MSSM plus an additional dimension-five operator obtained after integrating out the heavy fields needed to implement the seesaw of type I, II, or III:

$$W^{\rm MSSM,\nu} \ = \ W^{\rm MSSM} + W_\nu, \tag{E·28}$$

with

$$W^{\rm MSSM} \ = \ U^c Y_U Q H_u - D^c Y_D Q H_d - E^c Y_E L H_d + \mu H_u H_d, \tag{E·29}$$

and W_ν given in Eq. (2·39). The RGEs for the superpotential parameters are known, but we report them for completeness.

- Beta function coefficients: $b_i = \left\{\frac{33}{5}, 1, -3\right\}$.
- Yukawa couplings:

$$\begin{aligned}\dot{Y}_U &= \gamma_{U^c}^T Y_U + Y_U \gamma_Q + \gamma_{H_u} Y_U, \\ \dot{Y}_D &= \gamma_{D^c}^T Y_D + Y_D \gamma_Q + \gamma_{H_d} Y_D, \\ \dot{Y}_E &= \gamma_{E^c}^T Y_E + Y_E \gamma_L + \gamma_{H_d} Y_E,\end{aligned} \tag{E·30}$$

where the anomalous dimension matrices γ_Q, γ_{U^c}, etc. are

$$\begin{aligned}\gamma_Q &= -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{60}g_1^2\right)\mathbf{1} + Y_U^\dagger Y_U + Y_D^\dagger Y_D, \\ \gamma_{U^c} &= -2\left(\frac{4}{3}g_3^2 + \frac{4}{15}g_1^2\right)\mathbf{1} + 2Y_U^* Y_U^T, \\ \gamma_{D^c} &= -2\left(\frac{4}{3}g_3^2 + \frac{1}{15}g_1^2\right)\mathbf{1} + 2Y_D^* Y_D^T, \\ \gamma_L &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)\mathbf{1} + Y_E^\dagger Y_E, \\ \gamma_{E^c} &= -2\left(\frac{3}{5}g_1^2\right)\mathbf{1} + 2Y_E^* Y_E^T, \\ \gamma_{H_u} &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_U^\dagger Y_U\right), \\ \gamma_{H_d} &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right).\end{aligned} \tag{E·31}$$

- Superpotential dimensionful parameters:

$$\begin{aligned}\dot{\mu} &= (\gamma_{H_u} + \gamma_{H_d})\,\mu, \\ \dot{\kappa} &= \gamma_L^T \kappa + \kappa \gamma_L + 2\gamma_{H_u}\kappa.\end{aligned} \tag{E·32}$$

The SUSY-breaking terms completing the description of this model are collected in:

$$\widetilde{V}^{\mathrm{MSSM},\nu} = \widetilde{V}^{\mathrm{MSSM}} + \widetilde{V}_\nu, \tag{E·33}$$

where $\widetilde{V}^{\mathrm{MSSM}}$ is the usual

$$\begin{aligned}\widetilde{V}^{\mathrm{MSSM}} = &\left\{\widetilde{U}^c A_U\, \widetilde{Q} H_u - \widetilde{D}^c A_D\, \widetilde{Q} H_d - \widetilde{E}^c A_E\, \widetilde{L} H_d + B\, H_u H_d \;\; + \;\text{H.c.}\right\} \\ &+ \widetilde{Q}^*\, \widetilde{m}_Q^2\, \widetilde{Q} + \widetilde{U}^c\, \widetilde{m}_{U^c}^2\, \widetilde{U}^{c*} + \widetilde{D}^c\, \widetilde{m}_{D^c}^2\, \widetilde{D}^{c*} \\ &+ \widetilde{L}^*\, \widetilde{m}_L^2\, \widetilde{L} + \widetilde{E}^c\, \widetilde{m}_{E^c}^2\, \widetilde{E}^{c*} \\ &+ \widetilde{m}_{H_u}^2\, \widetilde{H}_u^* \widetilde{H}_u + \widetilde{m}_{H_d}^2\, \widetilde{H}_d^* \widetilde{H}_d \\ &+ \frac{1}{2}\left(M_3\, \widetilde{g}\widetilde{g} + M_2\, \widetilde{W}\widetilde{W} + M_1\, \widetilde{B}\widetilde{B}\right),\end{aligned} \tag{E·34}$$

and $\widetilde{V}_\nu$ is the nonrenormalizable lepton-number violating operator ($\Delta L = 2$)

$$\widetilde{V}_\nu = -\frac{1}{2}\widetilde{L}H_u\widetilde{\kappa}\widetilde{L}H_u. \tag{E·35}$$

The symmetric matrix $\widetilde{\kappa}$ has elements of $\mathcal{O}(\widetilde{m}/M_{\rm ssw})$, where $\widetilde{m}$ is a typical soft SUSY-breaking mass. We list in the following the RGEs for the parameters in this potential, except for those for the bi- and tri-linear scalar terms which are easily obtained by the algorithm mentioned above. We omit that for the dimensionless parameter $\widetilde{\kappa}$, which gives rise to a very suppressed bilinear term for the neutral component of $\widetilde{L}$.

- Soft sfermion masses:

$$\begin{aligned}
\dot{\widetilde{m}}^2_Q &= M^2_{G,Q} + \mathcal{F}_{(Y_U^\dagger,\widetilde{Q},\widetilde{U}^c,H_u,A_U^\dagger)} + \mathcal{F}_{(Y_D^\dagger,\widetilde{Q},\widetilde{D}^c,H_d,A_D^\dagger)},\\
\dot{\widetilde{m}}^2_{U^c} &= M^2_{G,U^c} + 2\mathcal{F}_{(Y_U,\widetilde{U}^c,\widetilde{Q},H_u,A_U)},\\
\dot{\widetilde{m}}^2_{D^c} &= M^2_{G,D^c} + 2\mathcal{F}_{(Y_D,\widetilde{D}^c,\widetilde{Q},H_d,A_D)},\\
\dot{\widetilde{m}}^2_L &= M^2_{G,L} + \mathcal{F}_{(Y_E^\dagger,\widetilde{L},\widetilde{E}^c,H_d,A_E^\dagger)},\\
\dot{\widetilde{m}}^2_{E^c} &= M^2_{G,E^c} + 2\mathcal{F}_{(Y_E,\widetilde{E}^c,\widetilde{L},H_d,A_E)}.
\end{aligned} \tag{E·36}$$

- Soft Higgs masses:

$$\begin{aligned}
\dot{\widetilde{m}}^2_{H_d} &= M^2_{G,H_d} + 3{\rm Tr}\mathcal{F}_{(Y_D^\dagger,H_d,\widetilde{D}^c,\widetilde{Q},A_D^\dagger)} + {\rm Tr}\mathcal{F}_{(Y_E^\dagger,H_d,\widetilde{E}^c,\widetilde{L},A_E^\dagger)},\\
\dot{\widetilde{m}}^2_{H_u} &= M^2_{G,H_u} + 3{\rm Tr}\mathcal{F}_{(Y_U^\dagger,H_u,\widetilde{U}^c,\widetilde{Q},A_U^\dagger)}.
\end{aligned} \tag{E·37}$$

E.2. $M_{\rm ssw} < Q < M_{\rm GUT}$

We distinguish the three cases in which the dimension-five operator in Eq. (E·28) is induced by heavy right-handed neutrino singlets, N^c, or by Higgs fields in the 15 and $\bar{15}$ representation of SU(5), 15_H and $\bar{15}_H$, or by three matter fields in the adjoint representation of SU(5), 24_M. Those presented here are not the most minimal implementations of the three types of seesaw mechanism, which strictly speaking, require only two SU(2) triplet Higgs fields T and $\bar{T}$ for the type II seesaw and three SU(2) fermion triplets W_M for the type III, if no embedding in SU(5) is needed. (See Sec. 2, where the relevant seesaw superpotentials are denoted by $W'_{\rm sssw\,i}$, with i =I, II, III to differentiate them from those presented here.) Here, we introduce all fields in the 15_H, $\bar{15}_H$ and in the 24_M representation of SU(5) to which T, $\bar{T}$ and W_M belong, respectively, and we denote the corresponding seesaw superpotentials by $W_{\rm sssw\,i}$ (i =I, II, III). Thus, the complete superpotentials in the three cases are:

$$W^{{\rm MSSM},i} = W^{\rm MSSM} + W_{{\rm ssw}\,i} \qquad (i = \text{I, II, III}), \tag{E·38}$$

with $W^{\rm MSSM}$ given in Eq. (E·29) and $W_{\rm ssw\,i}$ (i = I, II, III) to be specified in the following. Similarly, the soft SUSY-breaking part of the scalar potential is, in the three cases,

$$\widetilde{V}^{{\rm MSSM},i} = \widetilde{V}^{\rm MSSM} + \widetilde{V}_{{\rm ssw}\,i} \qquad (i = \text{I, II, III}), \tag{E·39}$$

with $\widetilde{V}^{\rm MSSM}$ given in Eq. (E·34), and $\widetilde{V}_{{\rm ssw}\,i}$ also to be specified.

E.2.1. MSSM,I

The RGEs for this model have been widely studied in this context after Ref. 18) appeared. (See for example Ref. 19).) Nevertheless, we list them here for completeness. The model is described by the superpotential $W^{\mathrm{MSSM,I}}$ with

$$W_{\mathrm{ssw\,I}} = N^c Y_N^{\mathrm{I}} L H_u + \frac{1}{2} N^c M_N N^c. \tag{E·40}$$

The RGEs for the superpotential parameters are as follows.

- Beta function coefficients: $\quad b_i^{\mathrm{I}} = \left\{ \frac{33}{5}, 1, -3 \right\}.$
- Yukawa couplings: To those in Eq. (E·30), the following RGE

$$\dot{Y}_N^{\mathrm{I}} = \gamma_{N^c}^T Y_N^{\mathrm{I}} + Y_N^{\mathrm{I}} \gamma_L + \gamma_{H_u} Y_N^{\mathrm{I}}, \tag{E·41}$$

must be added. Notice, however, that γ_{H_u} and γ_L are now modified by the presence of the operator $N^c Y_N^{\mathrm{I}} L H_u$. Therefore, the following anomalous dimensions

$$\begin{aligned} \gamma_L &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)\mathbf{1} + Y_E^\dagger Y_E + Y_N^{\mathrm{I}\,\dagger} Y_N^{\mathrm{I}}, \\ \gamma_{H_u} &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_U^\dagger Y_U + Y_N^{\mathrm{I}\,\dagger} Y_N^{\mathrm{I}}\right), \\ \gamma_{N^c} &= \ 2 Y_N^{\mathrm{I}\,*} Y_N^{\mathrm{I}\,T}, \end{aligned} \tag{E·42}$$

must be specified, whereas γ_Q, γ_{U^c}, γ_{D^c}, γ_{E^c}, γ_{H_d} remain as in Eq. (E·31).
- Superpotential dimensionful parameters:

$$\dot{M}_N = \gamma_{N^c}^T M_N + M_N \gamma_{N^c}. \tag{E·43}$$

The RGE for μ is as in Eq. (E·32), with γ_{H_u} given in Eq. (E·42).
The soft SUSY-breaking part of the scalar potential, $\widetilde{V}_{\mathrm{ssw\,I}}$, is given by

$$\widetilde{V}_{\mathrm{ssw\,I}} \ = \ \left\{ \widetilde{N}^c A_N^{\mathrm{I}}\, \widetilde{L} H_u + \frac{1}{2}\widetilde{N}^c B_N \widetilde{N}^c + \ \mathrm{H.c.} \right\} + \widetilde{N}^c\, \widetilde{m}_N^2\, \widetilde{N}^{c\,*}, \tag{E·44}$$

and the parameters in $\widetilde{V}^{\mathrm{MSSM,I}}$ obey the following RGEs.

- Soft sfermion masses: Of the RGEs in Eq. (E·36), only that for $\widetilde{m}_L^2$ gets modified by the presence of the operator $N^c Y_N^{\mathrm{I}} L H_u$, whereas the RGE for the soft contribution to the mass of $\widetilde{N}^c$ needs to be added:

$$\begin{aligned} \dot{\widetilde{m}}_L^2 &= M_{G,L}^2 + \mathcal{F}_{(Y_E^\dagger, \widetilde{L}, \widetilde{E}^c, H_d, A_E^\dagger)} + \mathcal{F}_{(Y_N^{\mathrm{I}\,\dagger}, \widetilde{L}, \widetilde{N}^c, H_u, A_N^{\mathrm{I}\,\dagger})}, \\ \dot{\widetilde{m}}_{N^c}^2 &= 2\mathcal{F}_{(Y_N^{\mathrm{I}}, \widetilde{N}^c, \widetilde{L}, H_u, A_N^{\mathrm{I}})}. \end{aligned} \tag{E·45}$$

- Soft Higgs masses:

$$\dot{\widetilde{m}}_{H_u}^2 \ = \ M_{G,H_u}^2 + 3\mathrm{Tr}\mathcal{F}_{(Y_U^\dagger, H_u, \widetilde{U}^c, \widetilde{Q}, A_U^\dagger)} + \ \mathrm{Tr}\mathcal{F}_{(Y_N^{\mathrm{I}\,\dagger}, H_u, \widetilde{N}^c, \widetilde{L}, A_N^{\mathrm{I}\,\dagger})}. \tag{E·46}$$

The RGE for $\widetilde{m}_{H_d}^2$ is as in Eq. (E·37).

E.2.2. MSSM,II

The RGEs for this model can also be found in Ref. 24). Some of our equations, however, differ from those reported there. We give a list of these equations at the end of this section. The part of the superpotential needed to specify this model at these energies is

$$\begin{aligned} W_{\mathrm{ssw\,II}} = & \frac{1}{\sqrt{2}} L Y_N^{\mathrm{II}} T L - D^c Y_{Q_{15}} L Q_{15} + \frac{1}{\sqrt{2}} D^c Y_S S D^c \\ & + \frac{1}{\sqrt{2}} \lambda_{\bar{T}} H_u \bar{T} H_u + \frac{1}{\sqrt{2}} \lambda_T H_d T H_d \\ & + M_T T \bar{T} + M_{Q_{15}} Q_{15} \bar{Q}_{15} + M_S S \bar{S}, \end{aligned} \tag{E·47}$$

where Y_N^{II} and Y_S are symmetric matrices, $\lambda_{\bar{T}}$ and λ_T, as well as M_T, M_Q, and M_S, complex numbers. The RGEs for the superpotential parameters are as follows.

- Beta function coefficients: $b_i^{\mathrm{II}} = \left\{ \frac{68}{5}, 8, 4 \right\}$.
- Yukawa couplings: The RGEs for Y_U, Y_D, and Y_E are as in Eq. (E·30). Those for Y_N^{II}, Y_S, and $Y_{Q_{15}}$, and the flavour-independent couplings $\lambda_{\bar{T}}$ and λ_T are

$$\begin{aligned} \dot{Y}_N^{\mathrm{II}} &= \gamma_L^T Y_N^{\mathrm{II}} + Y_N^{\mathrm{II}} \gamma_L + \gamma_T Y_N^{\mathrm{II}}, \\ \dot{Y}_{Q_{15}} &= \gamma_{D^c}^T Y_{Q_{15}} + Y_{Q_{15}} \gamma_L + \gamma_{Q_{15}} Y_{Q_{15}}, \\ \dot{Y}_S &= \gamma_{D^c}^T Y_S + Y_S \gamma_{D^c} + \gamma_S Y_S, \\ \dot{\lambda}_{\bar{T}} &= \left(2\gamma_{H_u} + \gamma_{\bar{T}}\right) \lambda_{\bar{T}}, \\ \dot{\lambda}_T &= \left(2\gamma_{H_d} + \gamma_T\right) \lambda_T. \end{aligned} \tag{E·48}$$

For the anomalous dimensions γ_Q, γ_{U^c}, γ_{E^c}, see Eq. (E·31), the others are

$$\begin{aligned} \gamma_{D^c} &= -2\left(\frac{4}{3}g_3^2 + \frac{1}{15}g_1^2\right)\mathbf{1} + 2Y_D^* Y_D^T + 2Y_{Q_{15}}^* Y_{Q_{15}}^T + 4Y_S^* Y_S^T, \\ \gamma_L &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right)\mathbf{1} + Y_E^\dagger Y_E + 3Y_{Q_{15}}^\dagger Y_{Q_{15}} + 3Y_N^{\mathrm{II}\,\dagger} Y_N^{\mathrm{II}}, \\ \gamma_{H_u} &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_U^\dagger Y_U\right) + 3|\lambda_{\bar{T}}|^2, \\ \gamma_{H_d} &= -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_D^\dagger Y_D + Y_E^\dagger Y_E\right) + 3|\lambda_T|^2, \\ \gamma_{Q_{15}} &= -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{60}g_1^2\right) + \mathrm{Tr}\left(Y_{Q_{15}}^\dagger Y_{Q_{15}}\right), \\ \gamma_{\bar{Q}_{15}} &= -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{1}{60}g_1^2\right), \\ \gamma_T &= -2\left(2g_2^2 + \frac{3}{5}g_1^2\right) + \mathrm{Tr}\left(Y_N^{\mathrm{II}\,\dagger} Y_N^{\mathrm{II}}\right) + |\lambda_T|^2, \\ \gamma_{\bar{T}} &= -2\left(2g_2^2 + \frac{3}{5}g_1^2\right) + |\lambda_{\bar{T}}|^2, \end{aligned}$$

$$\gamma_S = -2\left(\frac{10}{3}g_3^2 + \frac{4}{15}g_1^2\right) + \mathrm{Tr}\left(Y_S^\dagger Y_S\right),$$
$$\gamma_{\bar S} = -2\left(\frac{10}{3}g_3^2 + \frac{4}{15}g_1^2\right). \tag{E·49}$$

- Superpotential dimensionful parameters:

$$\dot M_T = (\gamma_T + \gamma_{\bar T})\, M_T,$$
$$\dot M_{Q_{15}} = \left(\gamma_{Q_{15}} + \gamma_{\bar Q_{15}}\right) M_{Q_{15}},$$
$$\dot M_S = (\gamma_S + \gamma_{\bar S})\, M_S. \tag{E·50}$$

The RGE for μ is as in Eq. (E·32).
The part of the scalar potential specific to this model, $\widetilde V_{\mathrm{ssw\,II}}$, to be specified in addition to $\widetilde V^{\mathrm{MSSM}}$ is:

$$\begin{aligned}\widetilde V_{\mathrm{ssw\,II}} = \Big\{ & \frac{1}{\sqrt2}\widetilde L A_N^{\mathrm{II}}\, T \widetilde L - \widetilde D^c A_{Q_{15}}\, \widetilde L Q_{15} + \frac{1}{\sqrt2}\widetilde D^c A_S\, S \widetilde D^c \\ & + \frac{1}{\sqrt2} A_{\lambda_{\bar T}} H_u \bar T H_u + \frac{1}{\sqrt2} A_{\lambda_T} H_d T H_d \\ & + B_T\, T\bar T + B_{Q_{15}}\, Q_{15}\bar Q_{15} + B_S\, S\bar S + \mathrm{H.c.}\Big\} \\ & + \widetilde m^2_{Q_{15}}\, Q_{15}^* Q_{15} + \widetilde m^2_{\bar Q_{15}}\, \bar Q_{15}^* \bar Q_{15} + \widetilde m_S^2\, S^* S + \widetilde m^2_{\bar S}\, \bar S^* \bar S \\ & + \widetilde m_T^2\, T^* T + \widetilde m^2_{\bar T}\, \bar T^* \bar T. \end{aligned} \tag{E·51}$$

Here, A_N^{II} and A_S are symmetric matrices, $A_{\lambda_{\bar T}}$, A_{λ_T}, and B_T, $B_{Q_{15}}$, B_S, complex numbers. The parameters in $\widetilde V^{\mathrm{MSSM,II}}$ satisfy the following RGEs.

- Soft sfermion masses:

$$\begin{aligned}\dot{\widetilde m}^2_{D^c} = {} & M^2_{G,D^c} + 2\mathcal F_{(Y_D,\widetilde D^c,\widetilde Q,H_d,A_D)} + 2\mathcal F_{(Y_{Q_{15}},\widetilde D^c,\widetilde L,Q_{15},A_{Q_{15}})} \\ & + 4\mathcal F_{(Y_S,\widetilde D^c,\widetilde D^{c\,*},S,A_S)}, \\ \dot{\widetilde m}^2_L = {} & M^2_{G,L} + \mathcal F_{(Y_E^\dagger,\widetilde L,\widetilde E^c,H_d,A_E^\dagger)} + 3\mathcal F_{(Y_{Q_{15}}^\dagger,\widetilde L,\widetilde D^c,Q_{15},A_{Q_{15}}^\dagger)} \\ & + 3\mathcal F_{(Y_N^{\mathrm{II}\,\dagger},\widetilde L,\widetilde L^*,T,A_N^{\mathrm{II}\,\dagger})}. \end{aligned} \tag{E·52}$$

The RGEs for $\widetilde m_Q^2$, $\widetilde m^2_{U^c}$, $\widetilde m^2_{E^c}$ are as in Eq. (E·36).

- Soft Higgs masses:

$$\begin{aligned}\dot{\widetilde m}^2_{H_d} = {} & M^2_{G,H_d} + 3\mathrm{Tr}\mathcal F_{(Y_D^\dagger,H_d,\widetilde D^c,\widetilde Q,A_D^\dagger)} + \mathrm{Tr}\mathcal F_{(Y_E^\dagger,H_d,\widetilde E^c,\widetilde L,A_E^\dagger)} \\ & + 3\mathcal F_{(\lambda_T,H_d,H_d,T,A_{\lambda_T})}, \\ \dot{\widetilde m}^2_{H_u} = {} & M^2_{G,H_u} + 3\mathrm{Tr}\mathcal F_{(Y_U^\dagger,H_u,\widetilde U^c,\widetilde Q,A_U^\dagger)} + 3\mathcal F_{(\lambda_{\bar T},H_u,H_u,\bar T,A_{\lambda_{\bar T}})}, \\ \dot{\widetilde m}^2_{Q_{15}} = {} & M^2_{G,Q_{15}} + \mathrm{Tr}\mathcal F_{(Y_{Q_{15}}^\dagger,Q_{15},\widetilde D^c,\widetilde L,A_{Q_{15}}^\dagger)}, \\ \dot{\widetilde m}^2_{\bar Q_{15}} = {} & M^2_{G,\bar Q_{15}}, \end{aligned}$$

$$\dot{\widetilde{m}}_T^2 = M_{G,T}^2 + \mathrm{Tr}\mathcal{F}_{(Y_N^{\mathrm{II}\,\dagger},T,\widetilde{L}^*,\widetilde{L},A_N^{\mathrm{II}\,\dagger})} + \mathcal{F}_{(\lambda_T,T,H_d,H_d,A_{\lambda_T})},$$
$$\dot{\widetilde{m}}_{\bar{T}}^2 = M_{G,\bar{T}}^2 + \mathcal{F}_{(\lambda_{\bar{T}},\bar{T},H_u,H_u,A_{\lambda_{\bar{T}}})},$$
$$\dot{\widetilde{m}}_S^2 = M_{G,S}^2 + \mathrm{Tr}\mathcal{F}_{(Y_S,S,\widetilde{D}^{c\,*},\widetilde{D}^c,A_S)},$$
$$\dot{\widetilde{m}}_{\bar{S}}^2 = M_{G,\bar{S}}^2. \tag{E·53}$$

Notice that in the contributions originated by flavour-independent operators the order in which the three masses squared appear in the function $\mathcal{F}$ is irrelevant. Both terms $\mathcal{F}_{(\lambda_T,H_d,H_d,T,A_{\lambda_T})}$ and $\mathcal{F}_{(\lambda_T,T,H_d,H_d,A_{\lambda_T})}$, indeed, reduce to $2|\lambda_T|^2\left(2\widetilde{m}_{H_d}^2+\widetilde{m}_T^2\right)+2|A_{\lambda_T}|^2$.

Our RGEs for this model differ from those reported in Ref. 24) for the following parameters: Y_N^{II}, M_T, $\widetilde{m}_T^2$, $\widetilde{m}_S^2$, $\widetilde{m}_{Q_{15}}^2$, A_N^{II}, and $A_{Q_{15}}$.

E.2.3. MSSM,III

The superpotential is still that of Eq. (E·38) with $i=\mathrm{III}$ and $W_{\mathrm{ssw\,III}}$ is

$$\begin{aligned}W_{\mathrm{ssw\,III}} = {} & H_uW_MY_N^{\mathrm{III}}L-\sqrt{\frac{6}{5}}\frac{1}{2}H_uB_MY_B^{\mathrm{III}}L+H_u\bar{X}_MY_{\bar{X}_M}D^c\\ & +\frac{1}{2}B_MM_{B_M}B_M+\frac{1}{2}G_MM_{G_M}G_M+\frac{1}{2}W_MM_{W_M}W_M\\ & +X_MM_{X_M}\bar{X}_M,\end{aligned} \tag{E·54}$$

where M_{B_M}, M_{G_M} and M_{W_M} are symmetric matrices. The RGEs for the superpotential parameters are as follows.

- Beta function coefficients: $b_i^{\mathrm{III}}=\left\{\frac{108}{5},16,12\right\}$.
- Yukawa couplings: Y_U, Y_D, and Y_E have RGEs listed in Eq. (E·30). Those for Y_N^{III}, Y_B^{III}, and $Y_{\bar{X}_M}$ are

$$\dot{Y}_N^{\mathrm{III}} = \gamma_{W_M}^TY_N^{\mathrm{III}}+Y_N^{\mathrm{III}}\gamma_L+\gamma_{H_u}Y_N^{\mathrm{III}},$$
$$\dot{Y}_B^{\mathrm{III}} = \gamma_{B_M}^TY_B^{\mathrm{III}}+Y_B^{\mathrm{III}}\gamma_L+\gamma_{H_u}Y_B^{\mathrm{III}},$$
$$\dot{Y}_{\bar{X}_M} = \gamma_{\bar{X}_M}^TY_{\bar{X}_M}+Y_{\bar{X}_M}\gamma_{D^c}+\gamma_{H_u}Y_{\bar{X}_M}. \tag{E·55}$$

For the anomalous dimensions γ_Q, γ_{U^c}, γ_{E^c}, γ_{H_d} see Eq. (E·31). The remaining ones are

$$\gamma_{D^c} = -2\left(\frac{4}{3}g_3^2+\frac{1}{15}g_1^2\right)\mathbf{1}+2Y_D^*Y_D^T+2Y_{\bar{X}_M}^\dagger Y_{\bar{X}_M},$$
$$\gamma_L = -2\left(\frac{3}{4}g_2^2+\frac{3}{20}g_1^2\right)\mathbf{1}+Y_E^\dagger Y_E+\frac{3}{2}Y_N^{\mathrm{III}\,\dagger}Y_N^{\mathrm{III}}+\frac{3}{10}Y_B^{\mathrm{III}\,\dagger}Y_B^{\mathrm{III}},$$
$$\gamma_{W_M} = -2\left(2g_2^2\right)\mathbf{1}+Y_N^{\mathrm{III}\,*}Y_N^{\mathrm{III}\,T},$$
$$\gamma_{B_M} = \frac{3}{5}Y_B^{\mathrm{III}\,*}Y_B^{\mathrm{III}\,T},$$

$$\gamma_{\bar{X}_M} = -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{5}{12}g_1^2\right)\mathbf{1} + Y^*_{\bar{X}_M} Y^T_{\bar{X}_M},$$
$$\gamma_{X_M} = -2\left(\frac{4}{3}g_3^2 + \frac{3}{4}g_2^2 + \frac{5}{12}g_1^2\right)\mathbf{1},$$
$$\gamma_{G_M} = -2\left(3g_3^2\right)\mathbf{1},$$
$$\gamma_{H_u} = -2\left(\frac{3}{4}g_2^2 + \frac{3}{20}g_1^2\right) + \mathrm{Tr}\left(3Y_U^\dagger Y_U + \frac{3}{2}Y_N^{\mathrm{III}\,\dagger} Y_N^{\mathrm{III}} + \frac{3}{10}Y_B^{\mathrm{III}\,\dagger} Y_B^{\mathrm{III}} + 3Y^\dagger_{\bar{X}_M} Y_{\bar{X}_M}\right). \quad \text{(E·56)}$$

- Superpotential dimensionful parameters:

$$\begin{aligned}
\dot{M}_{B_M} &= \gamma^T_{B_M} M_{B_M} + M_{B_M}\gamma_{B_M},\\
\dot{M}_{G_M} &= \gamma^T_{G_M} M_{G_M} + M_{G_M}\gamma_{G_M},\\
\dot{M}_{W_M} &= \gamma^T_{W_M} M_{W_M} + M_{W_M}\gamma_{W_M},\\
\dot{M}_{X_M} &= \gamma^T_{X_M} M_{X_M} + M_{X_M}\gamma_{\bar{X}_M}.
\end{aligned} \quad \text{(E·57)}$$

The term $\widetilde{V}_{\mathrm{ssw\,III}}$ specific of this model is

$$\begin{aligned}
\widetilde{V}_{\mathrm{ssw\,III}} = \Big\{ & H_u \widetilde{W}_M A_N^{\mathrm{III}} \widetilde{L} - \sqrt{\frac{6}{5}}\frac{1}{2} H_u \widetilde{B}_M A_B^{\mathrm{III}} \widetilde{L} + H_u \widetilde{\bar{X}}_M A_{\bar{X}_M} \widetilde{D}^c \\
& + \frac{1}{2}\widetilde{B}_M B_{B_M} \widetilde{B}_M + \frac{1}{2}\widetilde{G}_M B_{G_M} \widetilde{G}_M + \frac{1}{2}\widetilde{W}_M B_{W_M} \widetilde{W}_M \\
& + \widetilde{X}_M B_{X_M} \widetilde{\bar{X}}_M + \mathrm{H.c.}\Big\} \\
& + \widetilde{W}_M\, \widetilde{m}^2_{W_M}\, \widetilde{W}^*_M + \widetilde{B}_M\, \widetilde{m}^2_{B_M}\, \widetilde{B}^*_M + \widetilde{G}_M\, \widetilde{m}^2_{G_M}\, \widetilde{G}^*_M \\
& + \widetilde{X}_M\, \widetilde{m}^2_{X_M}\, \widetilde{X}^*_M + \widetilde{\bar{X}}_M\, \widetilde{m}^2_{\bar{X}_M}\, \widetilde{\bar{X}}^*_M.
\end{aligned} \quad \text{(E·58)}$$

The RGEs for the parameters in the complete soft scalar potential $\widetilde{V}^{\mathrm{MSSM,III}}$ are as follows.

- Soft sfermion masses:

$$\begin{aligned}
\dot{\widetilde{m}}^2_{D^c} &= M^2_{G,D^c} + 2\mathcal{F}_{(Y_D,\widetilde{D}^c,\widetilde{Q},H_d,A_D)} + 2\mathcal{F}_{(Y^T_{\bar{X}_M},\widetilde{D}^c,\widetilde{\bar{X}}^*_M,H_u,A^T_{\bar{X}_M})},\\
\dot{\widetilde{m}}^2_{L} &= M^2_{G,L} + \mathcal{F}_{(Y_E^\dagger,\widetilde{L},\widetilde{E}^c,H_d,A_E^\dagger)} + \frac{3}{2}\mathcal{F}_{(Y_N^{\mathrm{III}\,\dagger},\widetilde{L},\widetilde{W}_M,H_u,A_N^{\mathrm{III}\,\dagger})} \\
&\qquad + \frac{3}{10}\mathcal{F}_{(Y_B^{\mathrm{III}\,\dagger},\widetilde{L},\widetilde{B}_M,H_u,A_B^{\mathrm{III}\,\dagger})},\\
\dot{\widetilde{m}}^2_{W_M} &= M^2_{G,W_M} + \mathcal{F}_{(Y_N^{\mathrm{III}},\widetilde{W}_M,\widetilde{L},H_u,A_N^{\mathrm{III}})},\\
\dot{\widetilde{m}}^2_{B_M} &= \frac{3}{5}\mathcal{F}_{(Y_B^{\mathrm{III}},\widetilde{B}_M,\widetilde{L},H_u,A_B^{\mathrm{III}})},\\
\dot{\widetilde{m}}^2_{\bar{X}_M} &= M^2_{G,\bar{X}_M} + \mathcal{F}_{(Y_{\bar{X}_M},\widetilde{\bar{X}}_M,\widetilde{D}^{c\,*},H_u,A_{\bar{X}_M})},
\end{aligned}$$

$$\dot{\tilde{m}}^2_{X_M} = M^2_{G,X_M},$$
$$\dot{\tilde{m}}^2_{G_M} = M^2_{G,G_M}. \tag{E·59}$$

For $\dot{\tilde{m}}^2_Q$, $\dot{\tilde{m}}^2_{U^c}$, $\dot{\tilde{m}}^2_{E^c}$, see Eq. (E·36).

- Soft Higgs masses:

$$\begin{aligned}\dot{\tilde{m}}^2_{H_u} &= M^2_{G,H_u} + 3\mathrm{Tr}\mathcal{F}_{(Y_U^\dagger, H_u, \widetilde{U}^c, \widetilde{Q}, A_U^\dagger)} + \frac{3}{2}\mathrm{Tr}\mathcal{F}_{(Y_N^{\mathrm{III}\,\dagger}, H_u, \widetilde{W}_M, \widetilde{L}, A_N^{\mathrm{III}\,\dagger})} \\ &\quad + \frac{3}{10}\mathrm{Tr}\mathcal{F}_{(Y_B^{\mathrm{III}\,\dagger}, H_u, \widetilde{B}_M, \widetilde{L}, A_B^{\mathrm{III}\,\dagger})} + 3\mathrm{Tr}\mathcal{F}_{(Y_{\bar{X}_M}^\dagger, H_u, \widetilde{\bar{X}}_M, \widetilde{D}^{c\,*}, A_{\bar{X}_M}^\dagger)}.\end{aligned} \tag{E·60}$$

The RGE for $\widetilde{m}^2_{H_d}$ is as in Eq. (E·37).

E.3. $M_{\mathbf{GUT}} < Q$

We give here the RGEs for the minimal SUSY SU(5) model without NROs, with the three possible implementation of the seesaw mechanism. The mechanism's mediators are schematically denoted as RHNs, 15_{H}, and 24_{M} in three cases. The superpotentials and the soft SUSY-breaking scalar potentials for the three resulting models can be decomposed as:

$$W^{\mathrm{MSSU(5)},i} = W^{\mathrm{MSSU(5)}} + W_{\mathrm{i}},$$
$$\widetilde{V}^{\mathrm{MSSU(5)},i} = \widetilde{V}^{\mathrm{MSSU(5)}} + \widetilde{V}_{\mathrm{i}} \qquad (\mathrm{i} = \mathrm{RHN},\ 15\mathrm{H},\ 24\mathrm{M}), \tag{E·61}$$

where $W^{\mathrm{MSSU(5)}}$ and $\widetilde{V}^{\mathrm{MSSU(5)}}$ are given in Sec. 2 (see Eqs. (2·1), (2·2), (2·3), and Eqs. (2·16), (2·17), respectively), W_{i} and $\widetilde{V}_{\mathrm{i}}$ in Sec. 2.3 (see Eqs. (2·46) and (2·49)).

E.3.1. MSSU(5),RHN

The RGEs for this model can be found also in Ref. 19). The differences between our equations and those given in that paper are listed below. We refer the reader to Secs. 2 and 2.3 for the superpotential and soft SUSY-breaking potential of this model.

- Beta function coefficient: $b_5^{\mathrm{I}} = -3$.
- Yukawa couplings:

$$\begin{aligned}\dot{Y}^{10} &= \gamma^T_{10_M} Y^{10} + Y^{10}\gamma_{10_M} + \gamma_{5_H} Y^{10}, \\ \dot{Y}^{5} &= \gamma^T_{\bar{5}_M} Y^{5} + Y^{5}\gamma_{10_M} + \gamma_{\bar{5}_H} Y^{5}, \\ \dot{Y}^{\mathrm{I}}_N &= \gamma^T_{N^c} Y^{\mathrm{I}}_N + Y^{\mathrm{I}}_N\gamma_{\bar{5}_M} + \gamma_{5_H} Y^{\mathrm{I}}_N, \\ \dot{\lambda}_5 &= (\gamma_{5_H} + \gamma_{24_H} + \gamma_{\bar{5}_H})\lambda_5, \\ \dot{\lambda}_{24} &= 3\gamma_{24_H}\lambda_{24},\end{aligned} \tag{E·62}$$

with anomalous dimensions:

$$\gamma_{10_M} = -2\left(\frac{18}{5}g_5^2\right)\mathbf{1} + 3Y^{10\,\dagger}Y^{10} + 2Y^{5\,\dagger}Y^{5},$$
$$\gamma_{\bar{5}_M} = -2\left(\frac{12}{5}g_5^2\right)\mathbf{1} + 4Y^{5\,*}Y^{5\,T} + Y_N^{\mathrm{I}\,\dagger}Y_N^{\mathrm{I}},$$

$$
\begin{aligned}
\gamma_{N^c} &= \ \ 5Y_N^{\mathrm{I}*}Y_N^{\mathrm{I}T},\\
\gamma_{5_H} &= -2\left(\frac{12}{5}g_5^2\right)+\mathrm{Tr}\left(3Y^{10\,\dagger}Y^{10}+Y_N^{\mathrm{I}\,\dagger}Y_N^{\mathrm{I}}\right)+\frac{24}{5}|\lambda_5|^2,\\
\gamma_{\bar{5}_H} &= -2\left(\frac{12}{5}g_5^2\right)+\mathrm{Tr}\left(4Y^{5\,\dagger}Y^{5}\right)+\frac{24}{5}|\lambda_5|^2,\\
\gamma_{24_H} &= -2\left(5g_5^2\right)+|\lambda_5|^2+\frac{21}{20}|\lambda_{24}|^2.
\end{aligned}
\tag{E·63}
$$

The coefficient 21/20 in γ_{24_H} can be calculated by using the properties of the third order Casimir for adjoint representations reported, for example in Ref. 71).

- Superpotential dimensionful parameters:

$$
\begin{aligned}
\dot{M}_N &= \gamma_{N^c}^T M_N + M_N\gamma_{N^c},\\
\dot{M}_5 &= (\gamma_{5_H}+\gamma_{\bar{5}_H})M_5,\\
\dot{M}_{24} &= 2\gamma_{24_H}M_{24}.
\end{aligned}
\tag{E·64}
$$

- Soft sfermion masses:

$$
\begin{aligned}
\dot{\tilde{m}}^2_{10_M} &= M^2_{G,10_M}+3\mathcal{F}_{(Y^{10\,\dagger},\widetilde{10}_M,\widetilde{10}^*_M,5_H,A^{10\,\dagger})}+2\mathcal{F}_{(Y^{5\,\dagger},\widetilde{10}_M,\widetilde{\bar{5}}_M,\bar{5}_H,A^{5\,\dagger})},\\
\dot{\tilde{m}}^2_{\bar{5}_M} &= M^2_{G,\bar{5}_M}+4\mathcal{F}_{(Y^5,\widetilde{\bar{5}}_M,\widetilde{10}_M,\bar{5}_H,A^5)}+\mathcal{F}_{(Y_N^{\mathrm{I}T},\widetilde{\bar{5}}_M,\widetilde{N}^{c\,*},5_H,A_N^{\mathrm{I}T})},\\
\dot{\tilde{m}}^2_{N^c} &= \ \ 5\mathcal{F}_{(Y_N^{\mathrm{I}},\widetilde{N}^c,\widetilde{\bar{5}}^*_M,5_H,A_N^{\mathrm{I}})}.
\end{aligned}
\tag{E·65}
$$

- Soft Higgs masses:

$$
\begin{aligned}
\dot{\tilde{m}}^2_{5_H} &= M^2_{G,5_H}+3\mathrm{Tr}\mathcal{F}_{(Y^{10\,\dagger},5_H,\widetilde{10}^*_M,\widetilde{10}_M,A^{10\,\dagger})}\\
&\qquad +\mathrm{Tr}\mathcal{F}_{(Y_N^{\mathrm{I}\,\dagger},5_H,\widetilde{N}^c,\widetilde{\bar{5}}^*_M,A_N^{\mathrm{I}\dagger})}+\frac{24}{5}\mathcal{F}_{(\lambda_5,5_H,24_H,\bar{5}_H,A_{\lambda_5})},\\
\dot{\tilde{m}}^2_{\bar{5}_H} &= M^2_{G,\bar{5}_H}+4\mathrm{Tr}\mathcal{F}_{(Y^{5\,\dagger},\bar{5}_H,\widetilde{\bar{5}}_M,\widetilde{10}_M,A^{5\,\dagger})}+\frac{24}{5}\mathcal{F}_{(\lambda_5,\bar{5}_H,5_H,24_H,A_{\lambda_5})},\\
\dot{\tilde{m}}^2_{24_H} &= M^2_{G,24_H}+\mathcal{F}_{(\lambda_5,24_H,\bar{5}_H,5_H,A_{\lambda_5})}+\frac{21}{20}\mathcal{F}_{(\lambda_{24},24_H,24_H,24_H,A_{\lambda_{24}})}.
\end{aligned}
\tag{E·66}
$$

Our RGEs for this model differ from those in Ref. 19) for γ_{5_H} and $\tilde{\gamma}_{5_H}$. Moreover, the parameters λ_5 and λ_{24}, as well as A_{λ_5} and $A_{\lambda_{24}}$, are systematically taken as vanishing in the RGEs listed in that paper.

E.3.2. MSSU(5),15H

Also in this case, expressions for the superpotential and the soft SUSY-breaking potential can be found in Secs. 2 and 2.3.

- Beta function coefficient: $b_5^{\mathrm{II}}=4$.
- Yukawa couplings: The RGEs for Y^{10}, Y^5, λ_5, and λ_{24} can be read from Eq. (E·62), those for Y_N^{II}, λ_{15}, λ_U, λ_D are

$$
\dot{Y}_N^{\mathrm{II}} = \gamma_{\bar{5}_M}^T Y_N^{\mathrm{II}} + Y_N^{\mathrm{II}}\gamma_{\bar{5}_M} + \gamma_{15_H}Y_N^{\mathrm{II}},
$$

$$\begin{aligned}
\dot{\lambda}_{15} &= \left(\gamma_{15_H} + \gamma_{24_H} + \gamma_{\overline{15}_H}\right)\lambda_{15},\\
\dot{\lambda}_{U} &= \left(2\gamma_{5_H} + \gamma_{\overline{15}_H}\right)\lambda_{U},\\
\dot{\lambda}_{D} &= \left(2\gamma_{\bar{5}_H} + \gamma_{15_H}\right)\lambda_{D}.
\end{aligned} \tag{E·67}$$

Among the anomalous dimensions, only γ_{10_M} is as in Eq. (E·63). The others are

$$\begin{aligned}
\gamma_{\bar{5}_M} &= -2\left(\frac{12}{5}g_5^2\right)\mathbf{1} + 4Y^{5\,*}Y^{5\,T} + 6Y_N^{\mathrm{II}\,*}Y_N^{\mathrm{II}\,T},\\
\gamma_{5_H} &= -2\left(\frac{12}{5}g_5^2\right) + \mathrm{Tr}\left(3Y^{10\,\dagger}Y^{10}\right) + \frac{24}{5}|\lambda_5|^2 + 6|\lambda_U|^2,\\
\gamma_{\bar{5}_H} &= -2\left(\frac{12}{5}g_5^2\right) + \mathrm{Tr}\left(4Y^{5\,\dagger}Y^{5}\right) + \frac{24}{5}|\lambda_5|^2 + 6|\lambda_D|^2,\\
\gamma_{24_H} &= -2\left(5g_5^2\right) + |\lambda_5|^2 + \frac{21}{20}|\lambda_{24}|^2 + 7|\lambda_{15}|^2,\\
\gamma_{15_H} &= -2\left(\frac{28}{5}g_5^2\right) + \mathrm{Tr}\left(Y_N^{\mathrm{II}\,\dagger}Y_N^{\mathrm{II}}\right) + |\lambda_D|^2 + \frac{56}{5}|\lambda_{15}|^2,\\
\gamma_{\overline{15}_H} &= -2\left(\frac{28}{5}g_5^2\right) + |\lambda_U|^2 + \frac{56}{5}|\lambda_{15}|^2.
\end{aligned} \tag{E·68}$$

- Superpotential dimensionful parameters:

$$\dot{M}_{15} = (\gamma_{15_H} + \gamma_{\overline{15}_H})M_{15}. \tag{E·69}$$

The RGEs for M_5 and M_{24} are as in Eq. (E·64).

- Soft sfermion masses: The RGE for $\widetilde{m}^2_{10_M}$ is still as in Eq. (E·65), that for $\widetilde{m}^2_{\bar{5}_M}$ is

$$\dot{\widetilde{m}}^2_{\bar{5}_M} = M^2_{G,\bar{5}_M} + 4\mathcal{F}_{(Y^5,\widetilde{\bar{5}}_M,\widetilde{10}_M,\bar{5}_H,A^5)} + 6\mathcal{F}_{(Y_N^{\mathrm{II}},\widetilde{\bar{5}}_M,\widetilde{\bar{5}}^*_M,15_H,A_N^{\mathrm{II}})}. \tag{E·70}$$

- Soft Higgs masses:

$$\begin{aligned}
\dot{\widetilde{m}}^2_{5_H} &= M^2_{G,5_H} + 3\mathrm{Tr}\mathcal{F}_{(Y^{10\,\dagger},5_H,\widetilde{10}^*_M,\widetilde{10}_M,A^{10\,\dagger})}\\
&\qquad + \frac{24}{5}\mathcal{F}_{(\lambda_5,5_H,24_H,\bar{5}_H,A_{\lambda_5})} + 6\mathcal{F}_{(\lambda_U,5_H,\overline{15}_H,5_H,A_{\lambda_U})},\\
\dot{\widetilde{m}}^2_{\bar{5}_H} &= M^2_{G,\bar{5}_H} + 4\mathrm{Tr}\mathcal{F}_{(Y^{5\,\dagger},\bar{5}_H,\widetilde{\bar{5}}_M,\widetilde{10}_M,A^{5\,\dagger})}\\
&\qquad + \frac{24}{5}\mathcal{F}_{(\lambda_5,\bar{5}_H,5_H,24_H,A_{\lambda_5})} + 6\mathcal{F}_{(\lambda_D,\bar{5}_H,15_H,\bar{5}_H,A_{\lambda_D})},\\
\dot{\widetilde{m}}^2_{24_H} &= M^2_{G,24_H} + \mathcal{F}_{(\lambda_5,24_H,\bar{5}_H,5_H,A_{\lambda_5})}\\
&\qquad + \frac{21}{20}\mathcal{F}_{(\lambda_{24},24_H,24_H,24_H,A_{\lambda_{24}})} + 7\mathcal{F}_{(\lambda_{15},24_H,\overline{15}_H,15_H,A_{\lambda_{15}})},\\
\dot{\widetilde{m}}^2_{15_H} &= M^2_{G,15_H} + \mathrm{Tr}\mathcal{F}_{(Y_N^{\mathrm{II}\,\dagger},15_H,\widetilde{\bar{5}}_M,\widetilde{\bar{5}}^*_M,A_N^{\mathrm{II}\,\dagger})}\\
&\qquad + \mathcal{F}_{(\lambda_D,15_H,\bar{5}_H,\bar{5}_H,A_{\lambda_D})} + \frac{56}{5}\mathcal{F}_{(\lambda_{15},15_H,24_H,\overline{15}_H,A_{\lambda_{15}})},\\
\dot{\widetilde{m}}^2_{\overline{15}_H} &= M^2_{G,\overline{15}_H} + \mathcal{F}_{(\lambda_U,\overline{15}_H,5_H,5_H,A_{\lambda_U})} + \frac{56}{5}\mathcal{F}_{(\lambda_{15},\overline{15}_H,15_H,24_H,A_{\lambda_{15}})}.
\end{aligned} \tag{E·71}$$

E.3.3. MSSU(5),24M

The definitions of the superpotential and soft SUSY-breaking scalar potential can be found in Secs. 2 and 2.3.

- Beta function coefficient: $b_5^{\rm III} = 12.$
- Yukawa couplings:

$$\dot{Y}_N^{\rm III} = \gamma_{24_M}^T Y_N^{\rm III} + Y_N^{\rm III}\gamma_{\bar{5}_M} + \gamma_{5_H} Y_N^{\rm III},$$
$$\dot{Y}_{24_M}^x = \gamma_{24_M}^T Y_{24_M}^x + Y_{24_M}^x\gamma_{24_M} + \gamma_{24_H} Y_{24_M}^x. \tag{E·72}$$

The RGEs for Y_5, Y_{10}, λ_5 and λ_{24} are formally as in Eq. (E·62). The anomalous dimensions are now:

$$\gamma_{\bar{5}_M} = -2\left(\frac{12}{5}g_5^2\right)\mathbf{1} + 4Y^{5\,*}Y^{5\,T} + \frac{24}{5}Y_N^{\rm III\,\dagger}Y_N^{\rm III},$$
$$\gamma_{24_M} = -2\left(5g_5^2\right)\mathbf{1} + Y_N^{\rm III\,*}Y_N^{\rm III\,T} + \frac{21}{10}\left(Y_{24_M}^{S\,\dagger}Y_{24_M}^S\right) + \frac{5}{2}\left(Y_{24_M}^{A\,\dagger}Y_{24_M}^A\right),$$
$$\gamma_{5_H} = -2\left(\frac{12}{5}g_5^2\right) + {\rm Tr}\left(3Y^{10\,\dagger}Y^{10} + \frac{24}{5}Y_N^{\rm III\,\dagger}Y_N^{\rm III}\right) + \frac{24}{5}|\lambda_5|^2,$$
$$\gamma_{24_H} = -2\left(5g_5^2\right) + |\lambda_5|^2 + \frac{21}{20}|\lambda_{24}|^2 + {\rm Tr}\left(\frac{21}{20}Y_{24_M}^{S\,\dagger}Y_{24_M}^S + \frac{5}{4}Y_{24_M}^{A\,\dagger}Y_{24_M}^A\right), \tag{E·73}$$

and γ_{10_M} and $\gamma_{\bar{5}_H}$ are listed in Eq. (E·63).

- Superpotential dimensionful parameters:

$$\dot{M}_{24_M} = \gamma_{24_M}^T M_{24_M} + M_{24_M}\gamma_{24_M}. \tag{E·74}$$

The RGEs for M_5 and M_{24} can be found in Eq. (E·64).

- Soft sfermion masses:

$$\dot{\tilde{m}}_{\bar{5}_M}^2 = M_{G,\bar{5}_M}^2 + 4\mathcal{F}_{(Y^5,\widetilde{\bar{5}}_M,\widetilde{10}_M,\bar{5}_H,A^5)} + \frac{24}{5}\mathcal{F}_{(Y_N^{\rm III\,T},\widetilde{\bar{5}}_M,\widetilde{24}_M^*,5_H,A_N^{\rm III\,T})},$$
$$\dot{\tilde{m}}_{24_M}^2 = M_{G,24_M}^2 + \mathcal{F}_{(Y_N^{\rm III},\widetilde{24}_M,\widetilde{\bar{5}}_M^*,5_H,A_N^{\rm III})} + \frac{21}{10}\mathcal{F}_{(Y_{24_M}^S,\widetilde{24}_M,\widetilde{24}_M^*,24_H,A_{24_M}^S)}$$
$$+ \frac{5}{2}\mathcal{F}_{(Y_{24_M}^A,\widetilde{24}_M,\widetilde{24}_M^*,24_H,A_{24_M}^A)}. \tag{E·75}$$

The RGE for $\tilde{m}_{10_M}^2$ is as in Eq. (E·65).

- Soft Higgs masses:

$$\dot{\tilde{m}}_{5_H}^2 = M_{G,5_H}^2 + 3{\rm Tr}\mathcal{F}_{(Y^{10\,\dagger},5_H,\widetilde{10}_M^*,\widetilde{10}_M,A^{10\,\dagger})} + \frac{24}{5}{\rm Tr}\mathcal{F}_{(Y_N^{\rm III\,\dagger},5_H,\widetilde{24}_M,\widetilde{\bar{5}}_M^*,A_N^{\rm III\,\dagger})}$$
$$+ \frac{24}{5}\mathcal{F}_{(\lambda_5,5_H,24_H,\bar{5}_H,A_{\lambda_5})},$$
$$\dot{\tilde{m}}_{24_H}^2 = M_{G,24_H}^2 + \mathcal{F}_{(\lambda_5,24_H,\bar{5}_H,5_H,A_{\lambda_5})} + \frac{21}{20}{\rm Tr}\mathcal{F}_{(Y_{24_M}^{S\,\dagger},24_H,\widetilde{24}_M,\widetilde{24}_M^*,A_{24_M}^{S\,\dagger})}$$
$$+ \frac{5}{4}{\rm Tr}\mathcal{F}_{(Y_{24_M}^{A\,\dagger},24_H,\widetilde{24}_M,\widetilde{24}_M^*,A_{24_M}^{A\,\dagger})}$$
$$+ \frac{21}{20}\mathcal{F}_{(\lambda_{24},24_H,24_H,24_H,A_{\lambda_{24}})}. \tag{E·76}$$

The RGE for $\widetilde{m}^2_{5_H}$ is as in Eq. (E·66).

E.4. $\boldsymbol{M_{\rm GUT} < Q}$ - *nonvanishing NROs*

As explained in the text, we express the superpotential and scalar potential in terms of effective operators, with effective parameters indicated in boldface. A decomposition similar to that of Eq. (E·61) still holds:

$$\begin{aligned}\boldsymbol{W}^{\rm MSSU(5),\it i} &= \boldsymbol{W}^{\rm MSSU(5)} + \boldsymbol{W}_{\rm i},\\ \widetilde{\boldsymbol{V}}^{\rm MSSU(5),\it i} &= \widetilde{\boldsymbol{V}}^{\rm MSSU(5)} + \widetilde{\boldsymbol{V}}_{\rm i} \qquad ({\rm i} = {\rm RHN,\ 15H,\ 24M}).\end{aligned} \tag{E·77}$$

The various seesaw potentials $\boldsymbol{W}_{\rm i}$ and $\widetilde{\boldsymbol{V}}_{\rm i}$, (i = RHN, 15H, 24M) will be given explicitly in the following subsections. The two potentials $\boldsymbol{W}^{\rm MSSU(5)}$ and $\widetilde{\boldsymbol{V}}^{\rm MSSU(5)}$, now decomposed as:

$$\begin{aligned}\boldsymbol{W}^{\rm MSSU(5)} &= \boldsymbol{W}^{\rm MSSU(5)}_{\rm M} + W^{\rm MSSU(5)}_{\rm H},\\ \widetilde{\boldsymbol{V}}^{\rm MSSU(5)} &= \widetilde{\boldsymbol{V}}^{\rm MSSU(5)}_{\rm M} + \widetilde{V}^{\rm MSSU(5)}_{\rm H} + \widetilde{V}^{\rm MSSU(5)}_{\rm gaug},\end{aligned} \tag{E·78}$$

include NROs only in the matter parts, which are now

$$\begin{aligned}\boldsymbol{W}^{\rm MSSU(5)}_{M} = &-D^c\boldsymbol{Y}^5_D\ QH_d - E^c(\boldsymbol{Y}^5_E\ )^T LH_d - D^c\boldsymbol{Y}^5_{DU}U^cH^C_D - L\,\boldsymbol{Y}^5_{LQ}QH^C_D\\ &+U^c\boldsymbol{Y}^{10}_U QH_u + U^c\boldsymbol{Y}^{10}_{UE}E^cH^C_U + \frac{1}{2}Q\,\boldsymbol{Y}^{10}_{QQ}QH^C_U,\end{aligned} \tag{E·79}$$

and

$$\begin{aligned}\widetilde{\boldsymbol{V}}^{\rm MSSU(5)}_{M} = &\Big\{-\widetilde{D}^c\boldsymbol{A}^5_D\ \widetilde{Q}H_d - \widetilde{E}^c(\boldsymbol{A}^5_E\ )^T\widetilde{L}H_d - \widetilde{D}^c\boldsymbol{A}^5_{DU}\widetilde{U}^cH^C_D - \widetilde{L}\,\boldsymbol{A}^5_{LQ}\widetilde{Q}H^C_D\\ &+\widetilde{U}^c\boldsymbol{A}^{10}_U\widetilde{Q}H_u + \widetilde{U}^c\boldsymbol{A}^{10}_{UE}\widetilde{E}^cH^C_U + \frac{1}{2}\widetilde{Q}\,\boldsymbol{A}^{10}_{QQ}\widetilde{Q}H^C_U + {\rm H.c.}\Big\}\\ &+\widetilde{Q}^*\,\widetilde{\boldsymbol{m}}^2_Q\,\widetilde{Q} + \widetilde{U}^c\,\widetilde{\boldsymbol{m}}^2_{U^c}\,\widetilde{U}^{c\,*} + \widetilde{D}^c\,\widetilde{\boldsymbol{m}}^2_{D^c}\,\widetilde{D}^{c\,*}\\ &+\widetilde{L}^*\,\widetilde{\boldsymbol{m}}^2_L\,\widetilde{L} + \widetilde{E}^c\,\widetilde{\boldsymbol{m}}^2_{E^c}\,\widetilde{E}^{c\,*}.\end{aligned} \tag{E·80}$$

No boldface type is used for $\widetilde{V}^{\rm MSSU(5)}_{\rm gaug}$, which is still that of Eq. (2·17). Normal character types are also used for the Higgs potentials, $W^{\rm MSSU(5)}_H$ and $\widetilde{V}^{\rm MSSU(5)}_H$, because we neglect NROs in the Higgs sector. These potentials are then those in Eqs. (2·3) and (2·17). Similarly, also the purely Higgs parts of $\boldsymbol{W}_{15H}$ and $\widetilde{\boldsymbol{V}}_{15H}$ remain unchanged. The corresponding RGEs are also those reported in the case of vanishing NROs, with undecomposed anomalous dimensions, soft mass squared, trilinear and bilinear couplings. Nevertheless, we need, at least formally, to decompose these last quantities when they enter in RGEs of effective flavoured fields. In this case, we use distinct symbols, for example $\boldsymbol{\gamma}_{H_u}$ and $\boldsymbol{\gamma}_{H^C_U}$ for the same γ_{5_H}, as well as $\widetilde{\boldsymbol{m}}^2_{H_u}$ and $\widetilde{\boldsymbol{m}}^2_{H^C_U}$ for $\widetilde{m}^2_{5_H}$, but we make the identifications $\boldsymbol{\gamma}_{H_u} = \boldsymbol{\gamma}_{H^C_U} = \gamma_{5_H}$ and $\widetilde{\boldsymbol{m}}^2_{H_u} = \widetilde{\boldsymbol{m}}^2_{H^C_U} = \widetilde{m}^2_{5_H}$. As said, these are only formal decompositions, and we approximate the Yukawa couplings of the renormalizable operators, Y^{10}, Y^5, $Y^{\rm I}_N$,

which enter in their definitions or in the expressions of their RGEs, with the effective couplings $\boldsymbol{Y}_U^{10}$, $\boldsymbol{Y}_D^{5}$, and $\boldsymbol{Y}_N^{\mathrm{i}}$ (i = I,II,III), respectively.

We also neglect NROs for terms that involve only heavy fields, even when these have flavour, such as the fields 24_M. In the expressions for the anomalous dimensions of these fields, we use a "hybrid" form, with contributions from the undecomposed couplings of interactions for which we have omitted NROs (Higgs couplings), and from the decomposed effective couplings of interactions for which NROs are nonvanishing. (See for example Eq. (E·95).) A similar treatment in this section is reserved to the RGEs for the soft masses of such superheavy flavour fields. We keep undecomposed the contributions from the interactions that do not involve MSSM fields, but we do decompose the contributions from the flavour interactions to which the MSSM fields take part. (See for example Eq. (E·98).) This is to allow SU(5)-violating field rotations needed to embed the light fields in the SU(5) multiplets.

E.4.1. nrMSSU(5),RHN

The form of the superpotential $\boldsymbol{W}_{\mathrm{RHN}}$ is now

$$\boldsymbol{W}_{\mathrm{RHN}} = N^c \boldsymbol{Y}_N^{\mathrm{I}} L H_u - N^c \boldsymbol{Y}_{ND}^{\mathrm{I}} D^c H_U^C + \frac{1}{2} N^c M_N N^c. \tag{E·81}$$

The RGEs for the superpotential parameters in this class of models are as follows.

- Effective Yukawa couplings:

$$\begin{aligned}
\dot{\boldsymbol{Y}}_U^{10} &= \boldsymbol{\gamma}_{U^c}^T \boldsymbol{Y}_U^{10} + \boldsymbol{Y}_U^{10} \boldsymbol{\gamma}_{\boldsymbol{Q}} + \boldsymbol{\gamma}_{H_u} \boldsymbol{Y}_U^{10},\\
\dot{\boldsymbol{Y}}_{QQ}^{10} &= \boldsymbol{\gamma}_{Q}^T \boldsymbol{Y}_{QQ}^{10} + \boldsymbol{Y}_{QQ}^{10} \boldsymbol{\gamma}_{Q} + \boldsymbol{\gamma}_{H_U^C} \boldsymbol{Y}_{QQ}^{10},\\
\dot{\boldsymbol{Y}}_{UE}^{10} &= \boldsymbol{\gamma}_{U^c}^T \boldsymbol{Y}_{UE}^{10} + \boldsymbol{Y}_{UE}^{10} \boldsymbol{\gamma}_{E^c} + \boldsymbol{\gamma}_{H_U^C} \boldsymbol{Y}_{UE}^{10},\\
\dot{\boldsymbol{Y}}_D^{5} &= \boldsymbol{\gamma}_{D^c}^T \boldsymbol{Y}_D^{5} + \boldsymbol{Y}_D^{5} \boldsymbol{\gamma}_{Q} + \boldsymbol{\gamma}_{H_d} \boldsymbol{Y}_D^{5},\\
\dot{\boldsymbol{Y}}_E^{5} &= \boldsymbol{\gamma}_{L}^T \boldsymbol{Y}_E^{5} + \boldsymbol{Y}_E^{5} \boldsymbol{\gamma}_{E^c} + \boldsymbol{\gamma}_{H_d} \boldsymbol{Y}_E^{5},\\
\dot{\boldsymbol{Y}}_{DU}^{5} &= \boldsymbol{\gamma}_{D^c}^T \boldsymbol{Y}_{DU}^{5} + \boldsymbol{Y}_{DU}^{5} \boldsymbol{\gamma}_{U^c} + \boldsymbol{\gamma}_{H_D^C} \boldsymbol{Y}_{DU}^{5},\\
\dot{\boldsymbol{Y}}_{LQ}^{5} &= \boldsymbol{\gamma}_{L}^T \boldsymbol{Y}_{LQ}^{5} + \boldsymbol{Y}_{LQ}^{5} \boldsymbol{\gamma}_{Q} + \boldsymbol{\gamma}_{H_D^C} \boldsymbol{Y}_{LQ}^{5},
\end{aligned} \tag{E·82}$$

and

$$\begin{aligned}
\dot{\boldsymbol{Y}}_N^{\mathrm{I}} &= \boldsymbol{\gamma}_{N^c}^T \boldsymbol{Y}_N^{\mathrm{I}} + \boldsymbol{Y}_N^{\mathrm{I}} \boldsymbol{\gamma}_{L} + \boldsymbol{\gamma}_{H_u} \boldsymbol{Y}_N^{\mathrm{I}},\\
\dot{\boldsymbol{Y}}_{ND}^{\mathrm{I}} &= \boldsymbol{\gamma}_{N^c}^T \boldsymbol{Y}_{ND}^{\mathrm{I}} + \boldsymbol{Y}_{ND}^{\mathrm{I}} \boldsymbol{\gamma}_{D^c} + \boldsymbol{\gamma}_{H_U^C} \boldsymbol{Y}_{ND}^{\mathrm{I}}.
\end{aligned} \tag{E·83}$$

Given the approximation made here, the RGEs for λ_5 and λ_{24}, not decomposed, are as in Eq. (E·62). For the same reason, $\boldsymbol{\gamma}_{24_H}$, $\boldsymbol{\gamma}_{5_H}$ and $\boldsymbol{\gamma}_{\bar{5}_H}$ are those of Eq. (E·63), with Y^5, Y^{10}, and Y_N^{I} approximated by $\boldsymbol{Y}_D^{5}$, $\boldsymbol{Y}_U^{10}$, and $\boldsymbol{Y}_N^{\mathrm{I}}$, respectively. The anomalous dimensions $\boldsymbol{\gamma}_{H_u}$ and $\boldsymbol{\gamma}_{H_U^C}$ are taken to be equal to $\boldsymbol{\gamma}_{5_H}$; $\boldsymbol{\gamma}_{H_d}$ and $\boldsymbol{\gamma}_{H_D^C}$ to $\boldsymbol{\gamma}_{\bar{5}_H}$. The remaining anomalous dimensions needed for the evaluation of the above RGEs are

$$\gamma_Q = -2\left(\frac{18}{5} g_5^2\right)\mathbf{1} + \boldsymbol{Y}_U^{10\,\dagger} \boldsymbol{Y}_U^{10} + 2\boldsymbol{Y}_{QQ}^{10\,\dagger} \boldsymbol{Y}_{QQ}^{10} + \boldsymbol{Y}_D^{5\,\dagger} \boldsymbol{Y}_D^{5} + \boldsymbol{Y}_{LQ}^{5\,\dagger} \boldsymbol{Y}_{LQ}^{5},$$

$$
\begin{aligned}
\gamma_{U^c} &= -2\left(\frac{18}{5}g_5^2\right)\mathbf{1} + 2\boldsymbol{Y}_U^{10\,*}\boldsymbol{Y}_U^{10\,T} + \boldsymbol{Y}_{UE}^{10\,*}\boldsymbol{Y}_{UE}^{10\,T} + 2\boldsymbol{Y}_{DU}^{5\,\dagger}\boldsymbol{Y}_{DU}^{5},\\
\gamma_{E^c} &= -2\left(\frac{18}{5}g_5^2\right)\mathbf{1} + 3\boldsymbol{Y}_{UE}^{10\,\dagger}\boldsymbol{Y}_{UE}^{10} + 2\boldsymbol{Y}_E^{5\,\dagger}\boldsymbol{Y}_E^{5},\\
\gamma_{L} &= -2\left(\frac{12}{5}g_5^2\right)\mathbf{1} + \boldsymbol{Y}_E^{5\,*}\boldsymbol{Y}_E^{5\,T} + 3\boldsymbol{Y}_{LQ}^{5\,*}\boldsymbol{Y}_{LQ}^{5\,T} + \boldsymbol{Y}_N^{\mathrm{I}\,\dagger}\boldsymbol{Y}_N^{\mathrm{I}},\\
\gamma_{D^c} &= -2\left(\frac{12}{5}g_5^2\right)\mathbf{1} + 2\boldsymbol{Y}_D^{5\,*}\boldsymbol{Y}_D^{5\,T} + 2\boldsymbol{Y}_{DU}^{5\,*}\boldsymbol{Y}_{DU}^{5\,T} + \boldsymbol{Y}_{ND}^{\mathrm{I}\,\dagger}\boldsymbol{Y}_{ND}^{\mathrm{I}},\\
\gamma_{N^c} &= 2\boldsymbol{Y}_N^{\mathrm{I}\,*}\boldsymbol{Y}_N^{\mathrm{I}\,T} + 3\boldsymbol{Y}_{ND}^{\mathrm{I}\,*}\boldsymbol{Y}_{ND}^{\mathrm{I}\,T}.
\end{aligned}
\tag{E·84}
$$

- Superpotential dimensionful parameters: See Eq. (E·64). We are implicitly assuming that NROs involving only 24_H fields have small coefficients and/or that λ_{24} is not too small.

The term $\widetilde{\boldsymbol{V}}_{\mathrm{RHN}}$ specific of this model is

$$
\widetilde{\boldsymbol{V}}_{\mathrm{RHN}} = \left\{ N^c \boldsymbol{A}_N^{\mathrm{I}} L H_u - N^c \boldsymbol{A}_{ND}^{\mathrm{I}} D^c H_U^C + \frac{1}{2} N^c B_N N^c + \mathrm{H.c.}\right\} + \widetilde{N}^c\, \widetilde{\boldsymbol{m}}_{N^c}^2\, \widetilde{N}^{c\,*}.
\tag{E·85}
$$

The RGEs for the parameters appearing in $\widetilde{\boldsymbol{V}}^{\mathrm{MSSU(5),RHN}}$ are as follows.

- Soft sfermion masses:

$$
\begin{aligned}
\dot{\widetilde{\boldsymbol{m}}}{}_Q^2 &= M^2_{G,10_M} + \mathcal{F}_{(\boldsymbol{Y}_U^{10\,\dagger},\widetilde{\boldsymbol{Q}},\widetilde{\boldsymbol{U^c}},\boldsymbol{H_u},\boldsymbol{A}_U^{10\,\dagger})} + 2\mathcal{F}_{(\boldsymbol{Y}_{QQ}^{10\,\dagger},\widetilde{\boldsymbol{Q}},\widetilde{\boldsymbol{Q}}^*,\boldsymbol{H_U^C},\boldsymbol{A}_{QQ}^{10\,\dagger})}\\
&\quad + \mathcal{F}_{(\boldsymbol{Y}_D^{5\,\dagger},\widetilde{\boldsymbol{Q}},\widetilde{\boldsymbol{D^c}},\boldsymbol{H_d},\boldsymbol{A}_D^{5\,\dagger})} + \mathcal{F}_{(\boldsymbol{Y}_{LQ}^{5\,\dagger},\widetilde{\boldsymbol{Q}},\widetilde{\boldsymbol{L}}^*,\boldsymbol{H_D^C},\boldsymbol{A}_{LQ}^{5\,\dagger})},\\
\dot{\widetilde{\boldsymbol{m}}}{}_{U^c}^2 &= M^2_{G,10_M} + 2\mathcal{F}_{(\boldsymbol{Y}_U^{10},\widetilde{\boldsymbol{U^c}},\widetilde{\boldsymbol{Q}},\boldsymbol{H_u},\boldsymbol{A}_U^{10})} + \mathcal{F}_{(\boldsymbol{Y}_{UE}^{10},\widetilde{\boldsymbol{U^c}},\widetilde{\boldsymbol{E^c}}^*,\boldsymbol{H_U^C},\boldsymbol{A}_{UE}^{10})}\\
&\quad + 2\mathcal{F}_{(\boldsymbol{Y}_{DU}^{5\,T},\widetilde{\boldsymbol{U^c}},\widetilde{\boldsymbol{D^c}}^*,\boldsymbol{H_D^C},\boldsymbol{A}_{DU}^{5\,T})},\\
\dot{\widetilde{\boldsymbol{m}}}{}_{E^c}^2 &= M^2_{G,10_M} + 3\mathcal{F}_{(\boldsymbol{Y}_{UE}^{10\,T},\widetilde{\boldsymbol{E^c}},\widetilde{\boldsymbol{U^c}}^*,\boldsymbol{H_U^C},\boldsymbol{A}_{UE}^{10\,T})} + 2\mathcal{F}_{(\boldsymbol{Y}_E^{5\,T},\widetilde{\boldsymbol{E^c}},\widetilde{\boldsymbol{L}},\boldsymbol{H_d},\boldsymbol{A}_E^{5\,T})},\\
\dot{\widetilde{\boldsymbol{m}}}{}_L^2 &= M^2_{G,\bar{5}_M} + \mathcal{F}_{(\boldsymbol{Y}_E^{5\,*},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{E^c}},\boldsymbol{H_d},\boldsymbol{A}_E^{5\,*})} + 3\mathcal{F}_{(\boldsymbol{Y}_{LQ}^{5\,*},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{Q}}^*,\boldsymbol{H_D^C},\boldsymbol{A}_{LQ}^{5\,*})}\\
&\quad + \mathcal{F}_{(\boldsymbol{Y}_N^{\mathrm{I}\,\dagger},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{N^c}},\boldsymbol{H_u},\boldsymbol{A}_N^{\mathrm{I}\,\dagger})},\\
\dot{\widetilde{\boldsymbol{m}}}{}_{D^c}^2 &= M^2_{G,\bar{5}_M} + 2\mathcal{F}_{(\boldsymbol{Y}_D^{5},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{Q}},\boldsymbol{H_d},\boldsymbol{A}_D^{5})} + 2\mathcal{F}_{(\boldsymbol{Y}_{DU}^{5},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{U^c}}^*,\boldsymbol{H_D^C},\boldsymbol{A}_{DU}^{5})}\\
&\quad + \mathcal{F}_{(\boldsymbol{Y}_{ND}^{\mathrm{I}\,T},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{N^c}}^*,\boldsymbol{H_U^C},\boldsymbol{A}_{ND}^{\mathrm{I}\,T})},\\
\dot{\widetilde{\boldsymbol{m}}}{}_{N^c}^2 &= 2\mathcal{F}_{(\boldsymbol{Y}_N^{\mathrm{I}},\widetilde{\boldsymbol{N^c}},\widetilde{\boldsymbol{L}},\boldsymbol{H_u},\boldsymbol{A}_N^{\mathrm{I}})} + 3\mathcal{F}_{(\boldsymbol{Y}_{ND}^{\mathrm{I}},\widetilde{\boldsymbol{N^c}},\widetilde{\boldsymbol{D^c}}^*,\boldsymbol{H_U^C},\boldsymbol{A}_{ND}^{\mathrm{I}})},
\end{aligned}
\tag{E·86}
$$

  with $M^2_{G,10_M}$ and $M^2_{G,\bar{5}_M}$ as in the case of vanishing NROs.
- Soft Higgs masses: Since no decomposition is needed for 5_H, $\bar{5}_H$, and 24_H, the RGEs for $\widetilde{\boldsymbol{m}}^2_{5_H}$, $\widetilde{\boldsymbol{m}}^2_{\bar{5}_H}$, and $\widetilde{\boldsymbol{m}}^2_{24_H}$ are as in Eq. (E·66), with the already mentioned approximation taken for Y^5, Y^{10}, and Y_N^{I}.

E.4.2. nrMSSU(5),15H

The superpotential term $W_{\rm 15H}$ has now the form

$$\begin{aligned}\boldsymbol{W}_{\rm 15H} = \; & \frac{1}{\sqrt{2}} L \boldsymbol{Y}_N^{\rm II} T L - D^c \boldsymbol{Y}_{DL}^{\rm II} L Q_{15} + \frac{1}{\sqrt{2}} D^c \boldsymbol{Y}_{DD}^{\rm II} S D^c \\ & + \frac{1}{\sqrt{2}} \lambda_D \, \bar{5}_H 15_H \bar{5}_H + \frac{1}{\sqrt{2}} \lambda_U \, 5_H \overline{15}_H 5_H + \lambda_{15} 15_H 24_H \overline{15}_H \\ & + M_{15} 15_H \overline{15}_H, \end{aligned} \qquad (\text{E}\cdot 87)$$

where, only the first term in $W_{\rm 15H}$ of Eq. (2·46) was decomposed, with $Y_N^{\rm II}$ split in the three couplings $\boldsymbol{Y}_N^{\rm II}$, $\boldsymbol{Y}_{DL}^{\rm II}$, and $\boldsymbol{Y}_{DD}^{\rm II}$, The RGEs for the parameters in the superpotential are as follows.

- Effective Yukawa couplings:

$$\begin{aligned}\dot{\boldsymbol{Y}}_N^{\rm II} &= \boldsymbol{\gamma}_L^T \boldsymbol{Y}_N^{\rm II} + \boldsymbol{Y}_N^{\rm II} \gamma_L + \gamma_T \boldsymbol{Y}_N^{\rm II}, \\ \dot{\boldsymbol{Y}}_{DL}^{\rm II} &= \boldsymbol{\gamma}_{D^c}^T \boldsymbol{Y}_{DL}^{\rm II} + \boldsymbol{Y}_{DL}^{\rm II} \gamma_L + \gamma_{Q_{15}} \boldsymbol{Y}_{DL}^{\rm II}, \\ \dot{\boldsymbol{Y}}_{DD}^{\rm II} &= \boldsymbol{\gamma}_{D^c}^T \boldsymbol{Y}_{DD}^{\rm II} + \boldsymbol{Y}_{DD}^{\rm II} \gamma_{D^c} + \gamma_S \boldsymbol{Y}_{DD}^{\rm II}. \end{aligned} \qquad (\text{E}\cdot 88)$$

The RGEs for λ_{15}, λ_D and λ_U can be found in Eq. (E·67), the remaining ones in Eqs. (E·82). The anomalous dimensions $\boldsymbol{\gamma}_Q$, $\boldsymbol{\gamma}_{U^c}$, and $\boldsymbol{\gamma}_{E^c}$ are as in Eq. (E·84), $\boldsymbol{\gamma}_L$ and $\boldsymbol{\gamma}_{D^c}$ are

$$\begin{aligned}\gamma_L &= -2\left(\frac{12}{5} g_5^2\right) \boldsymbol{1} + \boldsymbol{Y}_E^{5\,*} \boldsymbol{Y}_E^{5\,T} + 3 \boldsymbol{Y}_{LQ}^{5\,*} \boldsymbol{Y}_{LQ}^{5\,T} + 3 \boldsymbol{Y}_N^{\rm II\,\dagger} \boldsymbol{Y}_N^{\rm II} + 3 \boldsymbol{Y}_{DL}^{\rm II\,\dagger} \boldsymbol{Y}_{DL}^{\rm II}, \\ \gamma_{D^c} &= -2\left(\frac{12}{5} g_5^2\right) \boldsymbol{1} + 2 \boldsymbol{Y}_D^{5\,*} \boldsymbol{Y}_D^{5\,T} + 2 \boldsymbol{Y}_{DU}^{5\,*} \boldsymbol{Y}_{DU}^{5\,T} + 4 \boldsymbol{Y}_{DD}^{\rm II\,*} \boldsymbol{Y}_{DD}^{\rm II\,T} \\ & \quad + 2 \boldsymbol{Y}_{DL}^{\rm II\,*} \boldsymbol{Y}_{DL}^{\rm II\,T}. \end{aligned} \qquad (\text{E}\cdot 89)$$

As mentioned above, $\boldsymbol{\gamma}_T$, $\boldsymbol{\gamma}_{Q_{15}}$, and $\boldsymbol{\gamma}_S$ are identified, $\boldsymbol{\gamma}_T = \boldsymbol{\gamma}_{Q_{15}} = \boldsymbol{\gamma}_S = \boldsymbol{\gamma}_{15_H}$, and the expression for $\boldsymbol{\gamma}_{15_H}$, $\boldsymbol{\gamma}_{\overline{15}_H}$, $\boldsymbol{\gamma}_{5_H}$, $\boldsymbol{\gamma}_{\bar{5}_H}$, and $\boldsymbol{\gamma}_{24_H}$ are as in Eq. (E·68).

- Superpotential dimensionful parameters: For $\dot{M}_{15}$, see Eq. (E·69); for $\dot{M}_5$ and $\dot{M}_{24}$, Eq. (E·64).

The term $\widetilde{\boldsymbol{V}}_{\rm 15H}$ specific of this model is

$$\begin{aligned}\widetilde{\boldsymbol{V}}_{\rm 15H} = \Big\{ & \frac{1}{\sqrt{2}} L \boldsymbol{A}_N^{\rm II} T L - D^c \boldsymbol{A}_{DL}^{\rm II} L Q_{15} + \frac{1}{\sqrt{2}} D^c \boldsymbol{A}_{DD}^{\rm II} S D^c \\ & + \frac{1}{\sqrt{2}} A_{\lambda_D} \bar{5}_H 15_H \bar{5}_H + \frac{1}{\sqrt{2}} A_{\lambda_U} 5_H \overline{15}_H 5_H + A_{\lambda_{15}} 15_H 24_H \overline{15}_H \\ & + B_{15} 15_H \overline{15}_H + \text{H.c.} \Big\} \\ & + \widetilde{m}_{15H}^2 \, 15_H^* 15_H + \widetilde{m}_{\overline{15}_H}^2 \, \overline{15}_H^* \overline{15}_H. \end{aligned} \qquad (\text{E}\cdot 90)$$

The RGEs for the parameters appearing in $\widetilde{\boldsymbol{V}}^{\rm MSSU(5),15H}$ are as follows.

- Soft sfermion masses:

$$\dot{\widetilde{\boldsymbol{m}}}^2_L = M^2_{G,\bar{5}_M} + \mathcal{F}_{(\boldsymbol{Y}^{5*}_E, \widetilde{\boldsymbol{L}}, \widetilde{\boldsymbol{E}^c}, \boldsymbol{H_d}, \boldsymbol{A}^{5,*}_E)} + 3\mathcal{F}_{(\boldsymbol{Y}^{5*}_{LQ}, \widetilde{\boldsymbol{L}}, \widetilde{\boldsymbol{Q}}^*, \boldsymbol{H^C_D}, \boldsymbol{A}^{5*}_{LQ})}$$
$$+ 3\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{II}\dagger}_N, \widetilde{\boldsymbol{L}}, \widetilde{\boldsymbol{L}}^*, \boldsymbol{T}, \boldsymbol{A}^{\mathrm{II}\dagger}_N)} + 3\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{II}\dagger}_{DL}, \widetilde{\boldsymbol{L}}, \widetilde{\boldsymbol{D}^c}, \boldsymbol{Q_{15}}, \boldsymbol{A}^{\mathrm{II}\dagger}_{DL})},$$
$$\dot{\widetilde{\boldsymbol{m}}}^2_{D^c} = M^2_{G,\bar{5}_M} + 2\mathcal{F}_{(\boldsymbol{Y}^5_D, \widetilde{\boldsymbol{D}^c}, \widetilde{\boldsymbol{Q}}, \boldsymbol{H_d}, \boldsymbol{A}^5_D)} + 2\mathcal{F}_{(\boldsymbol{Y}^5_{DU}, \widetilde{\boldsymbol{D}^c}, \widetilde{\boldsymbol{U}^c}^*, \boldsymbol{H^C_D}, \boldsymbol{A}^5_{DU})}$$
$$+ 4\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{II}}_{DD}, \widetilde{\boldsymbol{D}^c}, \widetilde{\boldsymbol{D}^c}^*, \boldsymbol{S}, \boldsymbol{A}^{\mathrm{II}}_{DD})} + 2\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{II}}_{DL}, \widetilde{\boldsymbol{D}^c}, \widetilde{\boldsymbol{L}}, \boldsymbol{Q_{15}}, \boldsymbol{A}^{\mathrm{II}}_{DL})}, \qquad (\mathrm{E{\cdot}91})$$

with $M^2_{G,\bar{5}_M}$ as in the case of vanishing NROs. The RGEs for $\widetilde{\boldsymbol{m}}^2_Q$, $\widetilde{\boldsymbol{m}}^2_{U^c}$, and $\widetilde{\boldsymbol{m}}^2_{E^c}$ are as in Eq. (E·86).

- Soft Higgs masses: See RGEs in Sec. E.3.2, with $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^{10}_U$ and $\boldsymbol{Y}^{\mathrm{II}}_N$ used for Y^5, Y^{10} and Y^{II}_N, respectively.

E.4.3. nrMSSU(5),24M

The missing ingredient needed to specify the superpotential of this class of models is

$$\boldsymbol{W}_{24\mathrm{M}} = H^C_U\Big(\sqrt{2}G_M\boldsymbol{Y}^{\mathrm{III}}_{GD} + \sqrt{\frac{6}{5}}\frac{1}{3}B_M\boldsymbol{Y}^{\mathrm{III}}_{BD}\Big)D^c + H^C_U X_M\boldsymbol{Y}^{\mathrm{III}}_{XL}L$$
$$+H_u\Big(\sqrt{2}W_M\boldsymbol{Y}^{\mathrm{III}}_N - \sqrt{\frac{6}{5}}\frac{1}{2}B_M\boldsymbol{Y}^{\mathrm{III}}_{BL}\Big)L + H_u\bar{X}_M\boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D}D^c$$
$$+\frac{1}{2}24_M M_{24_M}24_M + \frac{1}{2}\sum_{x=S,A}\big(24_M Y^x_{24_M}24_M\big)_x 24_H, \qquad (\mathrm{E{\cdot}92})$$

where only the first term in $W_{24\mathrm{M}}$ of Eq. (2·46) was decomposed, with Y^{III}_N split in the six couplings $\boldsymbol{Y}^{\mathrm{III}}_N$, $\boldsymbol{Y}^{\mathrm{III}}_{BL}$, $\boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D}$, and $\boldsymbol{Y}^{\mathrm{III}}_{GD}$, $\boldsymbol{Y}^{\mathrm{III}}_{BD}$, $\boldsymbol{Y}^{\mathrm{III}}_{XL}$. The index x, as usual, labels the symmetric and antisymmetric products of the two 24_M.

- Effective Yukawa couplings: Also in this case, the RGEs in Eq. (E·82) hold. The RGEs for λ_5, λ_{24} are in Eq. (E·62); those for the couplings $Y^x_{24_M}$ in Eq. (E·72). We have in addition:

$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_N = \boldsymbol{\gamma}^T_{W_M}\boldsymbol{Y}^{\mathrm{III}}_N + \boldsymbol{Y}^{\mathrm{III}}_N\boldsymbol{\gamma}_L + \boldsymbol{\gamma}_{H_u}\boldsymbol{Y}^{\mathrm{III}}_N,$$
$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_{BL} = \boldsymbol{\gamma}^T_{B_M}\boldsymbol{Y}^{\mathrm{III}}_{BL} + \boldsymbol{Y}^{\mathrm{III}}_{BL}\boldsymbol{\gamma}_L + \boldsymbol{\gamma}_{H_u}\boldsymbol{Y}^{\mathrm{III}}_{BL},$$
$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_{BD} = \boldsymbol{\gamma}^T_{B_M}\boldsymbol{Y}^{\mathrm{III}}_{BD} + \boldsymbol{Y}^{\mathrm{III}}_{BD}\boldsymbol{\gamma}_{D^c} + \boldsymbol{\gamma}_{H^C_U}\boldsymbol{Y}^{\mathrm{III}}_{BD},$$
$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_{GD} = \boldsymbol{\gamma}^T_{G_M}\boldsymbol{Y}^{\mathrm{III}}_{GD} + \boldsymbol{Y}^{\mathrm{III}}_{GD}\boldsymbol{\gamma}_{D^c} + \boldsymbol{\gamma}_{H^C_U}\boldsymbol{Y}^{\mathrm{III}}_{GD},$$
$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_{\bar{X}D} = \boldsymbol{\gamma}^T_{\bar{X}_M}\boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D} + \boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D}\boldsymbol{\gamma}_{D^c} + \boldsymbol{\gamma}_{H_u}\boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D},$$
$$\dot{\boldsymbol{Y}}^{\mathrm{III}}_{XL} = \boldsymbol{\gamma}^T_{X_M}\boldsymbol{Y}^{\mathrm{III}}_{XL} + \boldsymbol{Y}^{\mathrm{III}}_{XL}\boldsymbol{\gamma}_L + \boldsymbol{\gamma}_{H^C_U}\boldsymbol{Y}^{\mathrm{III}}_{XL}. \qquad (\mathrm{E{\cdot}93})$$

The anomalous dimensions $\boldsymbol{\gamma}_Q$, $\boldsymbol{\gamma}_{U^c}$ and $\boldsymbol{\gamma}_{E^c}$ are as in Eq. (E·84), $\boldsymbol{\gamma}_L$, $\boldsymbol{\gamma}_{D^c}$ are

$$\boldsymbol{\gamma}_L = -2\Big(\frac{12}{5}g_5^2\Big)\mathbf{1} + \boldsymbol{Y}^{5*}_E\boldsymbol{Y}^{5\,T}_E + 3\boldsymbol{Y}^{5*}_{LQ}\boldsymbol{Y}^{5\,T}_{LQ} + \frac{3}{2}\boldsymbol{Y}^{\mathrm{III}\dagger}_N\boldsymbol{Y}^{\mathrm{III}}_N$$

$$+\frac{3}{10}\boldsymbol{Y}_{BL}^{\mathrm{III}\,\dagger}\boldsymbol{Y}_{BL}^{\mathrm{III}}+3\boldsymbol{Y}_{XL}^{\mathrm{III}\,\dagger}\boldsymbol{Y}_{XL}^{\mathrm{III}},$$

$$\boldsymbol{\gamma}_{D^c}=-2\left(\frac{12}{5}g_5^2\right)\mathbf{1}+2\boldsymbol{Y}_D^{5\,*}\boldsymbol{Y}_D^{5\,T}+2\boldsymbol{Y}_{DU}^{5\,*}\boldsymbol{Y}_{DU}^{5\,T}+\frac{8}{3}\boldsymbol{Y}_{GD}^{\mathrm{III}\,\dagger}\boldsymbol{Y}_{GD}^{\mathrm{III}}$$
$$+\frac{2}{15}\boldsymbol{Y}_{BD}^{\mathrm{III}\,\dagger}\boldsymbol{Y}_{BD}^{\mathrm{III}}+2\boldsymbol{Y}_{\bar{X}D}^{\mathrm{III}\,\dagger}\boldsymbol{Y}_{\bar{X}D}^{\mathrm{III}}, \qquad \text{(E·94)}$$

with $\boldsymbol{\gamma}_{H_u}$, $\boldsymbol{\gamma}_{H_U^C}$, $\boldsymbol{\gamma}_{H_d}$, and $\boldsymbol{\gamma}_{H_D^C}$ are approximated as mentioned earlier. The anomalous dimensions for the fields that are components of 24_M can be written in the "hybrid" form:

$$\begin{aligned}
\boldsymbol{\gamma}_{G_M}&=-2\left(5g_5^2\right)\mathbf{1}+\frac{21}{10}Y_{24_M}^{S\,\dagger}Y_{24_M}^{S}+\frac{5}{2}Y_{24_M}^{A\,\dagger}Y_{24_M}^{A}+\boldsymbol{Y}_{GD}^{\mathrm{III}\,*}\boldsymbol{Y}_{GD}^{\mathrm{III}\,T},\\
\boldsymbol{\gamma}_{W_M}&=-2\left(5g_5^2\right)\mathbf{1}+\frac{21}{10}Y_{24_M}^{S\,\dagger}Y_{24_M}^{S}+\frac{5}{2}Y_{24_M}^{A\,\dagger}Y_{24_M}^{A}+\boldsymbol{Y}_{N}^{\mathrm{III}\,*}\boldsymbol{Y}_{N}^{\mathrm{III}\,T},\\
\boldsymbol{\gamma}_{B_M}&=-2\left(5g_5^2\right)\mathbf{1}+\frac{21}{10}Y_{24_M}^{S\,\dagger}Y_{24_M}^{S}+\frac{5}{2}Y_{24_M}^{A\,\dagger}Y_{24_M}^{A}+\frac{3}{5}\boldsymbol{Y}_{BL}^{\mathrm{III}\,*}\boldsymbol{Y}_{BL}^{\mathrm{III}\,T}\\
&\qquad+\frac{2}{5}\boldsymbol{Y}_{BD}^{\mathrm{III}\,*}\boldsymbol{Y}_{BD}^{\mathrm{III}\,T},\\
\boldsymbol{\gamma}_{X_M}&=-2\left(5g_5^2\right)\mathbf{1}+\frac{21}{10}Y_{24_M}^{S\,\dagger}Y_{24_M}^{S}+\frac{5}{2}Y_{24_M}^{A\,\dagger}Y_{24_M}^{A}+\boldsymbol{Y}_{XL}^{\mathrm{III}\,*}\boldsymbol{Y}_{XL}^{\mathrm{III}\,T},\\
\boldsymbol{\gamma}_{\bar{X}_M}&=-2\left(5g_5^2\right)\mathbf{1}+\frac{21}{10}Y_{24_M}^{S\,\dagger}Y_{24_M}^{S}+\frac{5}{2}Y_{24_M}^{A\,\dagger}Y_{24_M}^{A}+\boldsymbol{Y}_{\bar{X}D}^{\mathrm{III}\,*}\boldsymbol{Y}_{\bar{X}D}^{\mathrm{III}\,T}. \qquad \text{(E·95)}
\end{aligned}$$

- Superpotential dimensionful parameters: See Eq. (E·74) for $\dot{M}_{24_M}$, Eq. (E·64) for $\dot{M}_5$ and $\dot{M}_{24}$.

The term $\widetilde{\boldsymbol{V}}_{24\mathrm{M}}$ specific of this model is

$$\begin{aligned}
\widetilde{\boldsymbol{V}}_{24\mathrm{M}}=&\Bigg\{H_U^C\left(\sqrt{2}G_M\boldsymbol{A}_{GD}^{\mathrm{III}}+\sqrt{\frac{6}{5}}\frac{1}{3}B_M\boldsymbol{A}_{BD}^{\mathrm{III}}\right)D^c+H_U^CX_M\boldsymbol{A}_{XL}^{\mathrm{III}}L\\
&+H_u\left(\sqrt{2}W_M\boldsymbol{A}_N^{\mathrm{III}}-\sqrt{\frac{6}{5}}\frac{1}{2}B_M\boldsymbol{A}_{BL}^{\mathrm{III}}\right)L+H_u\bar{X}_M\boldsymbol{A}_{\bar{X}D}^{\mathrm{III}}D^c\\
&+\frac{1}{2}24_MB_{24_M}24_M+\frac{1}{2}\sum_{x=S,A}\left(24_MA_{24_M}^x24_M\right)_x24_H+\mathrm{H.c.}\Bigg\}\\
&+\widetilde{W}_M\,\widetilde{\boldsymbol{m}}_{W_M}^2\,\widetilde{W}_M^*+\widetilde{B}_M\,\widetilde{\boldsymbol{m}}_{B_M}^2\,\widetilde{B}_M^*\\
&+\widetilde{G}_M\,\widetilde{\boldsymbol{m}}_{G_M}^2\,\widetilde{G}_M^*+\widetilde{X}_M\,\widetilde{\boldsymbol{m}}_{X_M}^2\,\widetilde{X}_M^*+\widetilde{\bar{X}}_M\,\widetilde{\boldsymbol{m}}_{\bar{X}_M}^2\,\widetilde{\bar{X}}_M^*. \qquad \text{(E·96)}
\end{aligned}$$

The RGEs for the parameters appearing in $\widetilde{\boldsymbol{V}}^{\mathrm{MSSU(5),24M}}$ are as follows.

- Soft sfermion masses:

$$\dot{\widetilde{\boldsymbol{m}}}_L^2=M_{G,\bar{5}_M}^2+\mathcal{F}_{(\boldsymbol{Y}_E^{5\,*},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{E}^c},\boldsymbol{H_d},\boldsymbol{A}_E^{5\,*})}+3\,\mathcal{F}_{(\boldsymbol{Y}_{LQ}^{5\,*},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{Q}}^*,\boldsymbol{H_D^C},\boldsymbol{A}_{LQ}^{5\,*})}$$
$$+\frac{3}{2}\mathcal{F}_{(\boldsymbol{Y}_N^{\mathrm{III}\,\dagger},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{W}_M},\boldsymbol{H_u},\boldsymbol{A}_N^{\mathrm{III}\,\dagger})}+\frac{3}{10}\mathcal{F}_{(\boldsymbol{Y}_{BL}^{\mathrm{III}\,\dagger},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{B}_M},\boldsymbol{H_u},\boldsymbol{A}_{BL}^{\mathrm{III}\,\dagger})}$$

$$
\begin{aligned}
&+3\,\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}\dagger}_{XL},\widetilde{\boldsymbol{L}},\widetilde{\boldsymbol{X}_{\boldsymbol{M}}},\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}\dagger}_{XL})},\\
\dot{\widetilde{\boldsymbol{m}}}{}^2_D = {}& M^2_{G,\bar{5}_M} + 2\,\mathcal{F}_{(\boldsymbol{Y}^5_D,\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{Q}},\boldsymbol{H_d},\boldsymbol{A}^5_D)} + 2\,\mathcal{F}_{(\boldsymbol{Y}^5_{DU},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{U^c}}^*,\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{D}},\boldsymbol{A}^5_{DU})}\\
&+\frac{8}{3}\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}\,T}_{GD},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{G_M}}^*,\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}\,T}_{GD})} + \frac{2}{15}\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}\,T}_{BD},\widetilde{\boldsymbol{D^c}},\widetilde{\boldsymbol{B_M}}^*,\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}\,T}_{BD})}\\
&+2\,\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}\,T}_{\bar{X}D},\widetilde{\boldsymbol{D^c}},\widetilde{\bar{\boldsymbol{X}}_{\boldsymbol{M}}}^*,\boldsymbol{H_u},\boldsymbol{A}^{\mathrm{III}\,T}_{\bar{X}D})}. \qquad (\text{E}\cdot 97)
\end{aligned}
$$

The RGEs for $\widetilde{\boldsymbol{m}}^2_Q$, $\widetilde{\boldsymbol{m}}^2_{U^c}$, and $\widetilde{\boldsymbol{m}}^2_{E^c}$ are as in Eq. (E·86). As for the scalar components of the 24_M, their soft-mass RGEs can be given in the "hybrid" form:

$$
\begin{aligned}
\dot{\widetilde{\boldsymbol{m}}}{}^2_{G_M} &= (\widetilde{m}^{\mathrm{com}}_{24_M})^2 + \mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{GD},\widetilde{\boldsymbol{G_M}},\widetilde{\boldsymbol{D^c}}^*,\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}}_{GD})},\\
\dot{\widetilde{\boldsymbol{m}}}{}^2_{W_M} &= (\widetilde{m}^{\mathrm{com}}_{24_M})^2 + \mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{N},\widetilde{\boldsymbol{W_M}},\widetilde{\boldsymbol{L}},\boldsymbol{H_u},\boldsymbol{A}^{\mathrm{III}}_{N})},\\
\dot{\widetilde{\boldsymbol{m}}}{}^2_{B_M} &= (\widetilde{m}^{\mathrm{com}}_{24_M})^2 + \frac{3}{5}\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{BL},\widetilde{\boldsymbol{B_M}},\widetilde{\boldsymbol{L}},\boldsymbol{H_u},\boldsymbol{A}^{\mathrm{III}}_{BL})} + \frac{2}{5}\mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{BD},\widetilde{\boldsymbol{B_M}},\widetilde{\boldsymbol{D^c}}^*,\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}}_{BD})},\\
\dot{\widetilde{\boldsymbol{m}}}{}^2_{X_M} &= (\widetilde{m}^{\mathrm{com}}_{24_M})^2 + \mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{XL},\widetilde{\boldsymbol{X_M}},\widetilde{\boldsymbol{L}},\boldsymbol{H}^{\boldsymbol{C}}_{\boldsymbol{U}},\boldsymbol{A}^{\mathrm{III}}_{XL})},\\
\dot{\widetilde{\boldsymbol{m}}}{}^2_{\bar{X}_M} &= (\widetilde{m}^{\mathrm{com}}_{24_M})^2 + \mathcal{F}_{(\boldsymbol{Y}^{\mathrm{III}}_{\bar{X}D},\widetilde{\bar{\boldsymbol{X}}_{\boldsymbol{M}}},\widetilde{\boldsymbol{D^c}}^*,\boldsymbol{H_u},\boldsymbol{A}^{\mathrm{III}}_{\bar{X}D})}, \qquad (\text{E}\cdot 98)
\end{aligned}
$$

where the common term $(\widetilde{m}^{\mathrm{com}}_{24_M})^2$ is given by

$$
\begin{aligned}
(\widetilde{m}^{\mathrm{com}}_{24_M})^2 = M^2_{G,24_M} + \frac{21}{10}\mathcal{F}_{(Y^S_{24_M},\widetilde{24}^2_M,\widetilde{24}^{2\,*}_M,\widetilde{24}^2_H,A^S_{24_M})} &+\\
\frac{5}{2}\mathcal{F}_{(Y^A_{24_M},\widetilde{24}^2_M,\widetilde{24}^{2\,*}_M,\widetilde{24}^2_H,A^A_{24_M})}&. \qquad (\text{E}\cdot 99)
\end{aligned}
$$

The quantities $M^2_{G,\bar{5}_M}$ and $M^2_{G,24_M}$ are as in the case with vanishing NROs.

- Soft Higgs masses: The RGEs are as those given in Appendix E.3.3, with Y^5, Y^{10} and Y^{III}_N replaced by $\boldsymbol{Y}^5_D$, $\boldsymbol{Y}^{10}_U$ and $\boldsymbol{Y}^{\mathrm{III}}_N$, respectively.

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